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of the

Royal Astronomical Society

incorporating

the Geophysical Supplement to the Monthly Notices of the R. A. S.

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Finite Strain Theory and the Earth's Interior

J. F. Evernden

(Received 1957 February 18)

Summary

It is shown that the Poincaré-Brillouin approach to finite elastic strain theory leads to equations completely equivalent to those obtained by the Murnaghan-Birch approach. Applying the Brillouin pressure-strain equation to the Earth's interior (i.e. to Bullen's models), it is evident that there is a multiplicity of possible conclusions. The relative accuracy of the determined parameters is discussed and it is concluded that the Bullen data can be interpreted to mean a lower mantle with $\rho_0 = 4{\cdot}0$ g/cm^3, $K_0/\rho_0 = 33$ km^2/s^2, and an outer core having $\rho_0 = 6{\cdot}75$ g/cm^3, $K_0 = 1{\cdot}2 \times 10^{12}$ dyn/cm^2.

The problem of the composition of the Earth's interior from an elasticity point of view must be concerned with finite strain theory. However, the best method of applying this theory to a study of the Earth's interior may not be at all obvious. This uncertainty resides not in the theory but in the absence of experimental data on materials relevant to the problem, at relevant strains. As it turns out, assumptions necessarily made at the beginning of the analysis have a ruling influence on the conclusions drawn. The gross significance of these assumptions will be pointed out and it will be suggested that final conclusions are more reasonably based on derived values of ρ_0 and K_0 (zero pressure density and incompressibility, respectively) than on a derived value of $(dK/dp)_0$.

The basic assumptions that must be made are concerned with two problems: (A) the handling of third-order coefficients in equations such as (1) and (2) below. These are the equations relating stress to strain in the case of hydrostatic finite stress, (1) being taken from Birch (1938) and (2) from Brillouin (1925, 1946). A discussion of the equivalence of these two equations is in an appendix to this paper.

$$-p = \rho_0(1-2\epsilon)^{5/2}\left\{\frac{\partial\Phi}{\partial I_1} + 2\epsilon\frac{\partial\Phi}{\partial I_2} + \epsilon^2\frac{\partial\Phi}{\partial I_3}\right\} \qquad (1)$$

where

$$\left.\begin{aligned} \frac{\rho}{\rho_0} &= (1-2\epsilon)^{3/2} \\ \rho_0\Phi &= \tfrac{1}{2}(\lambda+2\mu)I_1^2-2\mu I_2+LI_1^3+MI_1I_2+NI_3+\ldots \\ \epsilon &= \beta-\tfrac{1}{2}\beta^2 \\ \beta &= \frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=\frac{\partial w}{\partial z} \\ I_1 &= 3\epsilon,\ I_2=3\epsilon^2,\ I_3=\epsilon^3, \end{aligned}\right\} \qquad (1a)$$

where x, y and z are Eulerian (i.e. reference coordinates are strained positions of points).

$$-p=\frac{1}{1+\alpha}\{\tfrac{1}{2}(3\lambda+2\mu)(2\alpha+\alpha^2)+2(27A+9B+3C)(2\alpha+\alpha^2)^2+\ldots\} \qquad (2)$$

where

$$\left.\begin{aligned} \frac{\rho}{\rho_0} &= \left(\frac{1}{1+\alpha}\right)^3 \\ \rho_0\,\Phi &= \tfrac{1}{8}\lambda K_1^2+\tfrac{1}{4}\mu K_2+AK_1K_2+BK_1^3+CK_3 \\ e &= 2\alpha+\alpha^2 \\ \alpha &= \frac{\partial u}{\partial a}=\frac{\partial v}{\partial b}=\frac{\partial w}{\partial c}, \end{aligned}\right.$$

where a, b and c are Lagrangian (i.e. reference coordinates are unstrained positions of points),

$$K_1=3e,\quad K_2=3e^2,\quad K_3=e^3.$$

(B) the starting point of the analysis or the data to be used in the analysis.

Considering the first problem, a possible assumption is that one is in a position to make an excellent estimate of the value of the third-order coefficients. Then such simplified theory plus seismic data will allow the computation of the second-order coefficients. These second-order coefficients are the values of the infinitesimal strain elastic parameters at zero pressure. A knowledge of these for the material of the Earth's interior may make it possible to draw some conclusions as to the nature of this material. This has been the approach taken by Birch (1951). He argues quite extensively the validity of the assumption that in equation (1), L, M and N can be considered as equal to zero for all materials. Equation (1) then simplifies to equation (3) (Birch 1952).

$$-p=3K_0\epsilon(1-2\epsilon)^{5/2}. \qquad (3)$$

This is equivalent to making $(dK/dp)_0$ identical for all materials and equal to 4. Birch marshals much data which purport to support this assumption, the data being computed values of $(dK/dp)_0$ which in turn are based on experimental values of volume changes under hydrostatic stress. Having assumed equation (3) to be valid, Birch then determines (K_0/ρ_0) for the material of the lower mantle, finds anomalous values, and concludes that there is a change of phase of the material of the Earth's mantle in the vicinity of Jeffreys's second-order discontinuity, this

change of phase being a response to increasing pressure downwards in the mantle.

Is the basic assumption warranted? In the first place, note that we have assumed that equation (3) is an excellent theoretical expression for the behaviour of all materials under stresses of the order of those encountered in the Earth. As the equation was derived by retaining only second-order terms in a power series expansion of stress vs. strain (rejecting all higher terms), with no concept of crystal or molecular structure other than isotropy entering the development, it would be surprising if equation (3) has such validity.

In the second place, values of $(dK/dp)_0$ can be found only by observations of volume vs. pressure at changing values of pressure, and by the fitting of such values to curves. The value of $(dK/dp)_0$ desired is effectively determined by the second derivative of this curve at zero pressure. The difficulty is that the curve fitting involves assumptions as to the analytical form of the expression to be used in the attempted fit. In other words, the form of the stress-strain equation must be assumed before the constants in the curve equation can be found. Since in the lower mantle we are probably concerned with olivine or similar material, it is important to realize that, by simply changing the form of the assumed V vs. P relationship, available data fit equally well curves with $(dK/dp)_0$ values of 2·1 and 15 (Verhoogen 1953). There is no basis for selecting one of these equations over the other and thus no basis for prescribing the "experimentally-derived" value of $(dK/dp)_0$ for olivine. Let it be noted here that the values of K_0 corresponding to the above values of $(dK/dp)_0$ are $1{\cdot}25 \times 10^{12}$ and $1{\cdot}16 \times 10^{12}$, respectively, illustrating the fact that lower order coefficients are more accurately determined from experimental data than are higher order coefficients and are much less sensitive to the equation being used.

As regards the second problem, a point in favour of the Birch analysis is that the data used are the seismic velocities, i.e. virtually directly observed data. The use of any other tabulated properties of the Earth's interior (pressure, density, etc.) rest upon the seismic data and assumptions as regards the behaviour of these other properties in the Earth or in given regions of the Earth's interior. Bullen (1953) has treated this problem extensively. Of interest here are his regions D and E, i.e., the lower mantle and the upper core. The derivation of density and pressure values for these regions involves assumptions as to stress pattern (hydrostatic) and temperature (adiabatic) in these regions and mass distributions in other portions of the Earth. The validity of these assumptions has been discussed extensively in the literature and there seems little doubt that the pressure and density values tabulated by Bullen for these two regions are within a very few per cent of the actual values and that use of them as starting point in a finite strain analysis is justified. As will be shown, it actually makes little difference in the final analysis whether the starting point data are density and pressure vs. depth or seismic velocity vs. depth.

Thus problem (1) is the critical one, and the one requiring careful consideration. Another possible approach to the analysis is to use a more general equation of state than that used by Birch. Thus an equation such as (1) or (2) would be superior to (3). If these equations are used as written (i.e. rejection of fourth-order terms), there are two unknowns in the equation of state for hydrostatic stress and use of simultaneous equations will give a solution for K_0 and $(dK/dp)_0$ as functions of ρ_0. On the basis of the discussion above, it would seem valid to attach greatest significance to the derived values of K_0 and ρ_0 and little to values of $(dK/dp)_0$ for two reasons:

i. we do not know what $(dK/dp)_0$ is for mantle material, and

ii. we are attempting to apply an imperfect formula to imperfect data and, under such conditions, must consider that we can make a better estimate of the lower order coefficients than of the higher.

In the calculations that follow, equation (2) will be used. The method used is to take Bullen's (1953) values of pressure and density at two points in a region of assumed homogeneity and to vary the zero-pressure value of the density (ρ_0) in order to obtain values for K_0 and $(dK/dp)_0$. An inspection of these values then indicates the appropriateness of the assumed values of ρ_0. It is evident from either equation (2) or (4) that H is probably negative for mantle materials.

Considering first the lower mantle, Bullen's values for depths of 1400 and 2898 km were used ($\rho_{1400} = 4{\cdot}92$ g/cm^3, $p_{1400} = 0{\cdot}58 \times 10^{12}$ dyn/cm^2, $\rho_{2898} = 5{\cdot}69$ g/cm^3, $p_{2898} = 1{\cdot}37 \times 10^{12}$ dyn/cm^2). Table 1 gives the results of computation for several values of ρ_0.

Table 1

ρ_0(g/cm^3)	H(10^{12} dyn/cm^2)	K_0(10^{12} dyn/cm^2)	K_0/ρ_0(km^2/s^2)	$(dK/dp)_0$
3·50	− 11·3	− 1·36	< 0	—
3·75	− 11·0	− ·298	< 0	—
3·95	− 10·4	+ ·753	19·06	24·7
4·00	− 10·0	+ 1·07	26·85	16·7
4·05	− 9·4	+ 1·46	36·03	11·4
4·25	− 6·51	+ 3·15	74·19	3·67
4·50	+ 5·69	+ 6·93	—	< 0

Table 2 shows the degree of agreement with Bullen's values at intermediate depths, using the parameters associated with $\rho_0 = 4{\cdot}00$ (p_t and p_B being theoretical and Bullen pressure values respectively).

Table 2

d(km)	ρ(g/cm^3)	p_t(10^{12} dyn/cm^2)	p_B(10^{12} dyn/cm^2)
1400	4·92	·580	·580
1800	5·12	·777	·780
2200	5·34	·989	·99
2600	5·54	1·204	1·20
2898	5·69	1·370	1·37

Table I indicates that a ρ_0 of less than 3·80 gives a negative K_0 and is thus inadmissible. Values of ρ_0 of greater than 4·35 give positive values of H and are thus unlikely. Values between these limits are theoretically admissible. Known values of K_0/ρ_0 for the materials of stony meteorites fall in the approximate range 25–35. A possible solution for the lower mantle would then be a ρ_0 of close to 4 g/cm^3 and a K_0/ρ_0 of close to 30 km^2/s^2. A density of 4·0 g/cm^2 appears to be a few per cent above that of stony meteorites. Reported densities of such meteorites are generally between 3·3 and 3·9 (Merrill 1909), those containing 15–20 per cent of free iron having densities of 3·60 to 3·65 (Clarke 1911). The difference between 3·6 and 4·0 may be due either to limitations of the analysis or to a real density differential, indicating a somewhat higher mean density for the Earth's lower mantle than for average stony meteorites.

Note that the $(dK/dp)_0$ value associated with $\rho_0 = 4{\cdot}0$ is 16·7, a value which in all probability is too high. Note also that the value of K_0/ρ_0 associated with

$(dK/dp)_0 = 4$ is 67–68. This is higher than the value of $K_0\rho_0$ found by Birch (i.e. 51) to be associated with a $(dK/dp)_0$ of 4, but, if Birch's line of analysis is followed, it also indicates the necessity of a phase change between 1400 km and the top of the mantle. Thus, Table 1 makes clear the fact that the results of applying finite strain theory to the mantle data depend upon an evaluation of the relative significance of quantities derived. The fact that the K_0 value known to be associated with supposed mantle-type material came out of the calculations associated with a density within several per cent of that of such material is thought to be highly significant and is given as the main result of this analysis.

Applying the same analysis to the outer core and using Bullen's (1953) values at depths of 2898 and 450 km (actually the mean of his two models), Table 3 is obtained ($\rho_{2898} = 9{\cdot}40$ g/cm^3, $p_{2898} = 1{\cdot}37 \times 10^{12}$ dyn/cm^2; $\rho_{4500} = 11{\cdot}20$ g/cm^3, $p_{4500} = 2{\cdot}81 \times 10^{12}$ dyn/cm^2).

Table 3

ρ_0(g/cm^3)	H(10^{12} dyn/cm^2)	K_0(10^{12} dyn/cm^2)	K_0/ρ_0(km^2/s^2)
6·25	—	< 0	—
6·50	− 12·3	+ 0·089	1·4
6·75	− 11·8	+ 0·995	14·7
7·00	− 10·9	+ 2·05	29·3
7·25	− 9·47	+ 3·26	45·0
7·75	− 4·02	+ 6·44	85·7
8·00	> 0		

Table 4 indicates the degree of agreement with Bullen's intermediate depth values (using $\rho_0 = 6{\cdot}75$).

Table 4

d(km)	ρ(g/cm^3)	p_I(10^{12} dyn/cm^2)	p_B(10^{12} dyn/cm^2)
2898	9·4	1·37	1·37
3000	9·55	1·48	1·47
3500	10·15	1·95	1·94
4000	10·70	2·39	2·40
4500	11·20	2·81	2·81
4982	11·50	3·06	3·17

It can be seen that a ρ_0 of between 6·5 and 7·85 is theoretically admissible. However, the value of $\rho_0 = 6{\cdot}75$ with a $K_0 = 1$ is in essentially perfect agreement with the estimated parameters of liquid iron at approximately 2500–3000° C, as we now show. It is to be noted that the elastic parameters tabulated by Bullen should be the adiabatic ones. Thus, the zero pressure parameters found here should be those for adiabatic release of pressure. With a core boundary at approximately 3000° C, adiabatic release of pressure should decrease the temperature by 500–1000° C. The density of liquid iron at 1550° C (melting point at zero pressure) is 7·0 – 7·2 g/cm^3. The estimated coefficient of thermal expansion is such as to lower this density to 6·6 – 6·7 at 2500°. Ten per cent of nickel would raise this density by 0·1 g/cm^3. The incompressibility of solid iron at zero pressure is $1{\cdot}6 \times 10^{12}$ dyn/cm^2, with the incompressibility of liquid iron being somewhat less. As can be seen from Table 3, a very small change in ρ_0 from 6·75 would make K_0 greater than 1·6.

Table 5 indicates the zero pressure parameters found by using Bullen's values of ρ associated with his hypothesis (ii) (Bullen 1953).

Table 5

ρ_0(g/cm^3)	H(10^{12} dyn/cm^2)	K_s(10^{12} dyn/cm^2)	K_s/ρ_s(km^2/s^2)
6·20	—	< 0	
6·35	− 12·9	0·117	
6·50	− 12·4	0·762	11·7
6·60	− 12·3	1·11	16·8

The most probable density is approximately 0·2 g/cm^3 less than for the previous case. Thus the data for the outer core are consistent with those to be expected of liquid iron at pressures present in the core.

In order to explain in detail the elastic parameter data of Bullen in these two regions, it is necessary to include an additional set of terms in the strain energy function beyond those necessary to explain the pressure data. The necessity for the inclusion of these terms is obvious as the elastic parameters are effectively derivatives of the pressure with respect to the strain.

Adopting the point of view suggested in this paper, it can be stated that the known data of the lower mantle and outer core are consistent with the assumption that these regions are composed of olivine orthosilicates plus 20–30 per cent free iron and liquid iron, respectively. No polymorphic phase changes are required to achieve this consistency.

The acceptance of the conclusion stated above, i.e. that available lower mantle data indicate that this region is composed of conventional stony meteoritic material under conditions of hydrostatic stress with an approximate adiabatic lapse rate, necessitates a very definite point of view as regards the interpretation of the known seismic velocities between 400 and 1000 km depth. These values are less than would be expected by extrapolation of the lower mantle curve of this paper into the upper mantle. Isostatic considerations indicate that a mantle density of 3·3 g/cm^3 is required. A decrease of ρ_0 upwards, probably associated with a decrease in free iron content, must take place above 100 km. All other factors remaining constant, this decrease in free iron content should lead to velocities greater than would be expected for lower mantle material at the same depth. Therefore the anomalous feature of the observed velocity vs. depth curves becomes the too rapid decrease of velocity upwards from approximately 1000 km. This feature of the results could be explained by appealing to a very marked departure from the adiabatic lapse rate in the upper mantle.

Thus the anomalous behaviour of the velocity curves in the region 400–1200 km, which Birch interpreted as implying a gross phase change below approximately 1000 km, is here interpreted as indicating a region of abnormally low rigidity (i.e. low velocities) above 1000 km. It is to be hoped that further investigation will be able to establish the ultimate status of these two possible interpretations.

Appendix

Some comments relative to the formula used for computation may be in order. This equation was presented by Brillouin (1925, 1946). Doubt as to the validity of his derivation arose due to an apparent misinterpretation of his nomenclature. It is, however, correct, and a simple process of substitution leads directly from the

equation (2) of Birch to equation (1) of Brillouin. It should be pointed out that the major differences in the Birch and Brillouin developments reside in the line of argument followed and not in the coordinates in which the equations are expressed. Birch has shown the equivalence of the two coordinate systems. No significance is to be attached to the coordinate system except in so far as it affects the exact values of the elastic coefficients. The values of K_0 and $(dK/dp)_0$ are not a function of the coordinate system used. However, due to apparently common misinterpretation of the Brillouin equation, it seems worthwhile to make the proof of the equivalence of these two equations (Birch and Brillouin) a matter of record. The equivalence will be shown for the case of finite hydrostatic strain.

Start with equation (1) and (1a) of this paper (Birch 1938). The relations between the Eulerian and Lagrangian strain invariants are given by Murnaghan (1937). By use of these relations, and by substituting (1a) into (1), we obtain, retaining terms in $(2\alpha+\alpha)^2$,

$$-p = \frac{1}{(1+\alpha)}\left\{\frac{3\lambda_1+2\mu_1}{2}(2\alpha+\alpha^2)+\tfrac{1}{4}(27l+9m+n)(2\alpha+\alpha^2)^2\right\} \qquad \text{(A1)}$$

where

$$\rho_0\Phi = \frac{\lambda_1+2\mu_1}{2}J_1^2-2\mu_1 J_2+lJ_1^3+mJ_1J_2+nJ_3$$

$$l = L-2(\lambda+2\mu),\quad m = M+4(\lambda+3\mu),\quad n = N-12\mu,\quad \lambda_1 = \lambda,\quad \mu_1 = \mu$$

$$J_1 = \frac{3e}{2},\quad J_2 = \frac{3e^2}{2}\quad J_3 = \frac{e^3}{3},\quad e = 2\alpha+\alpha^2.$$

Brillouin (1946) chose as the strain invariants

$$K_1 = \Sigma e_{ii},\quad K_2 = \Sigma l_{ik}l_{ki},\quad K_3 = \Sigma l_{ik}l_{ki}l_{ii},\quad \delta_0$$

$$J_1 = \frac{K_1}{2},\quad J_2 = \frac{1}{4}\frac{K_1^2}{2}-\frac{K_2}{2},\quad J_3 = \frac{1}{8}\frac{K_1^3}{6}-\frac{K_1K_2}{2}+\frac{K_3}{3}$$

with a resulting

$$\rho_0\Phi = \tfrac{1}{8}\lambda K_1^2+\tfrac{1}{4}\mu K_2+AK_1K_2+BK_1^2+CK_3.$$

Substitution and identification of corresponding coefficients show that

$$\tfrac{1}{4}(27l+9m+n) = 2(27A+9B+3C)$$

so that

$$-p = \frac{1}{(1+\alpha)}\left\{\frac{3\lambda+2\mu}{2}(2\alpha+\alpha^2)+2(27A+8B+3C)(2\alpha+\alpha^2)^2+\ldots\right\} \qquad \text{(A2)}$$

This is Brillouin's equation (x.106) and equation (2) of this paper.

Using relation (2) and defining the compressibility as $K = \rho dp/d\rho$, we obtain

$$K = \tfrac{1}{3}\{p+(1+\alpha)[3K_0+8H(2\alpha+\alpha^2)+6R(2\alpha+\alpha^2)^2+\ldots]\} \qquad \text{(A3)}$$

where $3K_0 = 3\lambda+2\mu$, $H = 27A+9B+3C$, and R is concerned with the

fourth-order terms in the strain. The (dK/dp) equation in Brillouin's nomenclature would be

$$\frac{dK}{dp} = \frac{1}{9K_0}\{p-(1+\alpha)^3[16H+24R(2\alpha+\alpha^2)+\ldots]\}$$

$$\rightarrow -\frac{16H}{9K_0} \text{ as } \alpha \rightarrow 0. \tag{A4}$$

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Diffracted Seismic waves near the PKP Caustic

K. E. Bullen and T. N. Burke-Gaffney

(Received 1957 August 26)

Summary

An examination is made of arrival-times of P' waves in the range $137° < \Delta < 142°$, as recorded from four hydrogen bomb explosions of 1954. Early readings in this range are attributed to diffracted *PKP* waves near the caustic at 142°. The results supply a small final link in the chain of evidence for the existence of the inner core. The possibility of detecting diffracted *PKP* waves from studies of natural earthquakes is also examined. The results illustrate the potentiality of controlled large explosions in advancing knowledge of the Earth's deep interior.

1. Introduction

In a previous paper (Burke-Gaffney & Bullen 1957), to be referred to as Paper I, a series of seismological readings of four hydrogen bombs exploded in the vicinity of Bikini atoll in 1954 was analysed. Certain of the readings bear interestingly (Bullen & Burke-Gaffney 1957) on the problem of diffraction near the *PKP* caustic at 142°, and the purpose of the present paper is to discuss the details, with special reference to the structure of the Earth's core.

Our previous analysis indicated a pattern in the exploding of the bombs, from which we inferred that an assumed epicentre of 11° 35′ N, 165° 30′ E (agreeing with that for the atom bomb explosion (Gutenberg & Richter 1946) of 1946 July 24 is substantially correct, and from which we inferred the origin-times to be:

I	1954	Feb	28d	18h	45m	0s
II	1954	Mar	26d	18h	30m	0s
III	1954	Apr	25d	18h	10m	0s
IV	1954	May	4d	18h	10m	0s

The reliability of conclusions to be drawn from the seismic data depends of course on the accuracy of these surmised origin-times, but we gave reasons in Paper I for believing that the accuracy is within about 0·2s. (The official times have not, to our knowledge, yet been released.)

2. P′ readings from the hydrogen bombs for $137° < \Delta < 142°$

The seismic data listed in Paper I give 36 readings of P or P' reported at stations for which the epicentral distance Δ is less than 120°, and 11 readings at stations for which $137° < \Delta < 142°$. These readings are repeated in Table 1. Paper I gives the corresponding residuals against the J.-B. tables for a surface focus.

Table 1

Times and nature of onsets of hydrogen bomb explosions

Station	(I) 28.ii.1954			(II) 26.iii.1954			(III) 25.iv.1954			(IV) 4.v.1954		
	min	s		min	s		min	s		min	s	
Noumea	51	45	(e)							16	45	(e)
Matsushiro	57	29	(e)	36	53	(e)						
Brisbane	52	43	(i)	37	43	(i)	17	43	(i)	17	43	(i)
Baguio	53	5	(i)	38	2	(e)	17	58	(e)	18	2	(e)
Riverview	53	35	(i)	38	36	(i)	18	35	(i)	18	36	(i)
Kaimata										19	28	(e)
Victoria	56	5	(i)	41	5	(i)				21	7	(e)
Pasadena	56	29	(i)	41	29	(i)	21	29	(i)	21	28	(i)
Fayetteville	58	8	(i)	43	9	(i)						
Quetta	58	6	(i)				23	8	(e)	23	7	(i)
Kiruna										23	30	(e)
Uppsala										24	12	(e)
Ksara				49	42	(e)						
Stuttgart	63	44	(e)	48	45	(e)	28	45	(e)	28	45	(e)
Tananarive				48	54	(e)				28	54	(i)
Pretoria	64	16	(i)	49	16	(e)				29	15	(e)
Kimberley	64	22	(e)	49	32	(e)	29	25	(e)	29	20	(e)
Tamanrasset	64	26	(e)	49	24	(e)	29 (traces)	28		29	28	(e)

For $\Delta < 120°$, the numerically greatest negative residual at all stations except Baguio ($\Delta = 43°{\cdot}8$) was found in Paper I to be $-4{\cdot}2$s. The mean of the 36 residuals is $-1{\cdot}9 \pm 1{\cdot}7$s. If the residuals at Baguio and also Uppsala—which, at $\Delta = 104°{\cdot}2$, alone has a large positive residual ($+4{\cdot}9$s)—are excluded, the mean is $-1{\cdot}5 \pm 1{\cdot}3$s.

The stations for which $\Delta > 120°$ are Pretoria ($137°{\cdot}4$), Kimberley ($139°{\cdot}6$) and Tamanrasset ($140°{\cdot}6$). A striking feature of the eleven residuals at these three stations was that (excepting one reading at Kimberley, later revised) all were appreciably less than $-4{\cdot}2$s. In the case of Pretoria, where only the explosions I, II and IV were recorded (the record of III was obscured by a smudge), the residuals were very self-consistent, being $-10{\cdot}7$, $-10{\cdot}7$ and $-11{\cdot}7$s. This feature was noteworthy at the preliminary stage when our data consisted entirely of routine observatory readings. At many observatories, including Pretoria and Kimberley, it had not been suspected that the readings listed in Paper I related to the hydrogen bomb explosions, or that the readings for the different explosions were in any way connected with one another. Thus there is unusual objectivity in the gathering of the data.

Had the readings for $\Delta > 120°$ been late instead of early, or not repeated so consistently as at Pretoria, they would have been simply explained. But because they were early, we decided to write to Professor A. L. Hales, Director of the Bernard Price Institute, who kindly re-examined the South African records, and gave us the following summary:

Pretoria. I: first movement at $-11{\cdot}7$s, and small; followed by another phase starting at $-3{\cdot}2$s, and a large movement at $-0{\cdot}4$s.

II: first movement not well defined, probably starting at $-10{\cdot}9$s, but definitely not later than $-9{\cdot}2$s; later phase with large movement at $+1{\cdot}3$s.

IV: first movement, small but clear at $-10{\cdot}6$s; large movement at $+1{\cdot}4$s.

Kimberley. I: first movement at −8·0s, but not well defined and possibly earlier; second phase at −1·6s, and large movement at +1·1s.

II: first movement at −8·0s; large movement at +1·0s.

IV: first movement at −10·7s, within about 0·5s; large movement at +1·8s.

As already pointed out, the shock III, which was generally the weakest recorded of the series, was not readable at Pretoria; on the Kimberley record of III, Dr Hales found a first movement apparently starting at −6·1s, and certainly definite 0·5s later. He states that there could be errors up to 3s in the readings of II, but that the Pretoria readings of IV are reliable within 1s. He also comments on the remarkable similarity of appearance of the Pretoria records of I, II and IV.

The outstanding result of this re-examination is that not merely are the early onsets well confirmed, but that there appear to be at least two distinct onsets on practically every occasion. There are sizable onsets at both Pretoria and Kimberley from I, II and IV, all within ±2s of the J.-B. *PKIKP* times; and these onsets are preceded by smaller movements starting some 8 to 12s earlier. Certain of the South African records are reproduced in Figures 1 and 2.

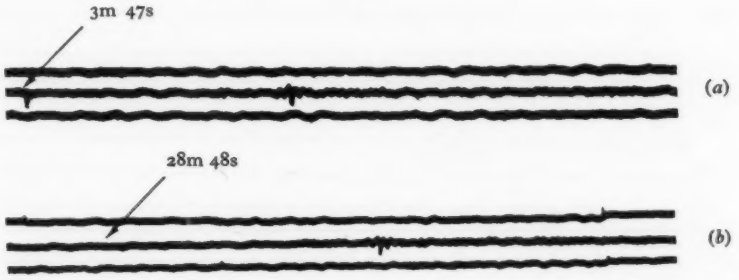

FIG. 1.—*Pretoria records for the explosions of* (*a*) 1954 *Feb.* 28, (*b*) 1954 *May* 4. *The time-marks are at half-minute intervals.*

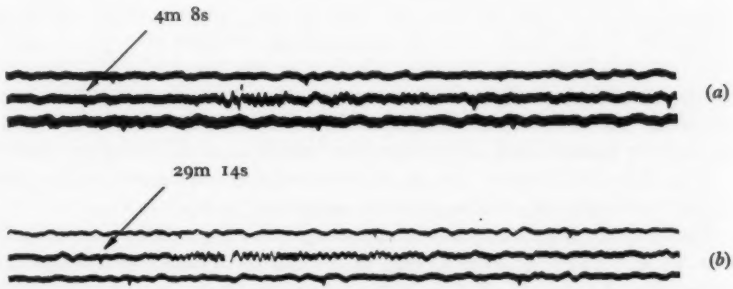

FIG. 2.—*Kimberley records for the explosions of* (*a*) 1954 *Feb.* 28, (*b*) 1954 *May* 4. *The time-marks are at half-minute intervals.*

Records taken at Tamanrasset were also kindly sent to us by Mlle. A. Grandjean, Director of the Tamanrasset Observatory, and are reproduced in Figure 3. Each of the four negative residuals shown in Paper I is associated with an *emersio* onset and there is a distinctly larger movement starting several seconds later. In each case the uncertainty in determining the time of the first onset is about 3s.

The early smaller movements at Pretoria, Kimberley and Tamanrasset could conceivably be associated with shock waves arising because of special conditions at

the source. In that event, negative residuals of magnitudes comparable with those at the three stations would also be expected at a number of stations recording the normal *P* phase. As already pointed out, it was, however, only at Baguio that such residuals had been indicated in the routine readings; (the values were $-4{\cdot}5$, $-7{\cdot}5$, $-11{\cdot}5$ and $-7{\cdot}5$s for the four shocks). Dr J. J. Hennessey, Director of the Baguio Observatory, has since kindly sent us the original records, most of which

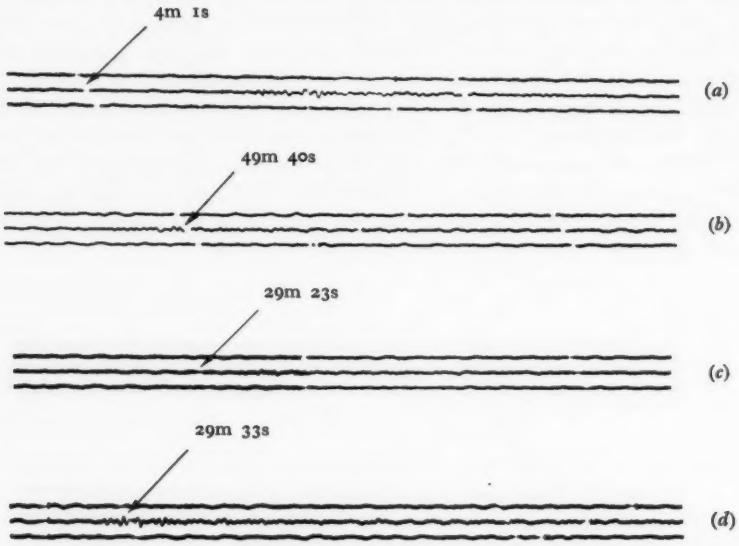

FIG. 3.—*Tamanrasset records for the explosions of* (*a*) 1954 *Feb.* 28, (*b*) 1954 *Mar.* 26, (*c*) 1954 *Apr.* 25, (*d*) 1954 *May* 4.

are very clear. The *Z*-component records are reproduced in Figure 4. A careful re-examination of the Baguio records gave revised residuals of $-3\frac{1}{2}$, $-3\frac{1}{2}$, $-3\frac{1}{2}$ and $-3\frac{1}{2}$s. (In particular, the reading which gave the residual of $-11{\cdot}5$s for shock III was definitely found to need a correction of several seconds.) The mean of the residuals is thus reduced in magnitude, and their self-consistency is complete. Although the mean is still $-3\frac{1}{2}$s, it is now within 2s of the mean for other stations for which $\Delta < 120°$. Hence it seems that some other explanation than shock waves is required for the early readings in the range $137° < \Delta < 142°$.

The likely explanation is that the readings arise from diffraction near the *PKP* caustic at $\Delta = 142°$. Jeffreys (1939), in the course of deciding whether the waves now considered to be *PKIKP* could have been associated with this caustic, showed that, for a harmonic wave train of period τ, the amplitude near the caustic should be proportional to the Airy integral

$$\int_0^\infty \cos\{\tfrac{1}{2}\pi(w^3 + mw)\}\,dx,$$

where $m = 1{\cdot}4(142° - \Delta)^{-2/3}$ and $x = 8{\cdot}5\, w\tau^{1/3}$. He showed that the integral has its first maximum for $\Delta < 142°$ at $m = -1{\cdot}1$, and that, as Δ decreases, the value falls to 0·01 of the maximum at $m = 3{\cdot}3$. The periods of the earlier arriving waves at

Pretoria and Kimberley are of the order of a second, so that observable diffracted waves could certainly be expected in the range $139° < \Delta < 142°$. Kimberley and Tamanrasset lie in fact inside this range. Pretoria lies outside the range, but only by 1°·6, and this is probably unimportant since the theory used by Jeffreys is a simple steady-state theory. The seismograms reproduced in Figures 1 and 2 show moreover that, in the case of the shocks recorded at Pretoria, the Pretoria amplitudes are less than the Kimberley ones.

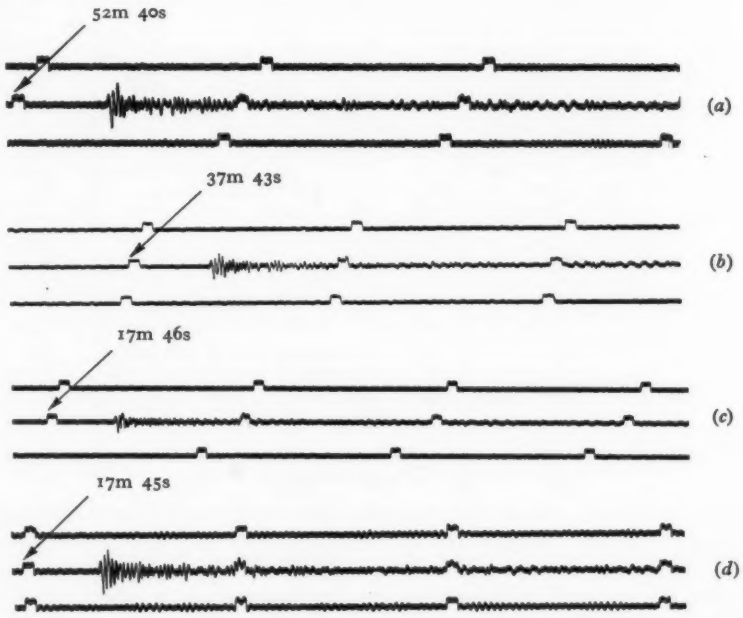

FIG. 4.—*Baguio records for the explosions of* (*a*) 1954 *Feb.* 28, (*b*) 1954 *Mar.* 26, (*c*) 1954 *Apr.* 25, (*d*) 1954 *May* 4.

The arrival-times of the first recorded movements at Pretoria, Kimberley and Tamanrasset are also in reasonable accord with an extension of the travel-time curve back from the *PKP* cusp. This is seen from Figure 5, which exhibits the J.-B. travel-time curves for a surface focus over the range $137° < \Delta < 144°$. In Figure 5, the means of the first arrival-times at the three stations are denoted by small circles. The mean times of the later onsets are shown as crosses, and fit well the expected times for the phase *PKIKP*.

Special significance attaches to these results (assuming our surmised epicentre and origin-times to be fairly correct) since bomb explosions, unlike natural earthquakes, can permit the precise calculation of travel-times. In addition, there are in this instance four shocks from the one centre yielding remarkably similar records and very self-consistent residuals. The evidence thus provided for the existence of diffracted *PKP* waves is therefore of special importance.

3. Early P′ readings from natural earthquakes for $137° < \Delta < 142°$

An examination of natural earthquake data was next made to see how far evidence supporting that in Section 2 might be found. Inspection of the data in the

I.S.S. showed that there appear in fact to be quite a number of early P' readings in the range $137° < \Delta < 142°$, but epicentral and origin-time uncertainties make it difficult to find cases which can be regarded as significant.

An interesting case found was a study made by Morita (1937) of Japanese readings of the Upper Amazon deep-focus earthquake of 1933 August 29. Morita gives 45 Japanese readings of P' in the range $138° < \Delta < 150°$. (Only eleven of these readings are included in the I.S.S.). The readings derive their importance from the

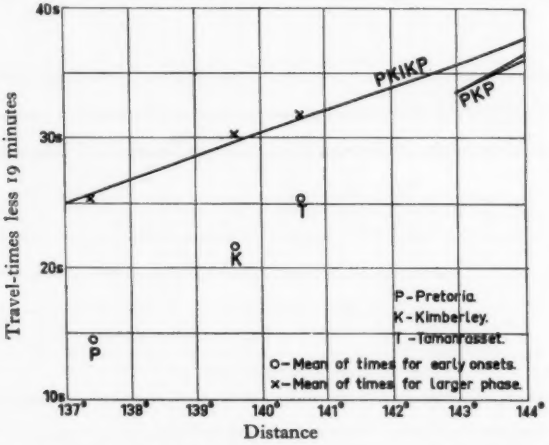

FIG. 5.—*Mean times of onset at Pretoria, Kimberley and Tamanrasset, shown against the J.-B. curves for a surface focus.*

fact that there are so many of them within a short range of epicentral distance and of azimuth. Thus any change in the estimated epicentre or origin-time would require the whole set of readings to be shifted as a block, without rotation, relative to co-ordinate systems such as those displayed in Figures 6 and 7. Also, corrections for ellipticity would affect all the readings equally. Hence the readings have the potentiality of giving reliable evidence on travel-time gradients.

Morita, who used Brunner's chart, states that his observed times are "earlier than Brunner's by as much as 8s at 140° and 12s at 139° when the times are adjusted to be equal to Brunner's at 144°. . . Question arises as to the evident steepness of the time-curve in the region $\Delta < 144°$. The slope of the observed curve in this region deviates too widely from that of Brunner's to look upon it for error's sake. Accordingly little doubts are left for its reality." Morita's investigation was carried out before he was aware of Lehmann's work (1936) on the phase *PKIKP*, and considerably before the final J.-B. tables were published. Thus he was of course unaware of the full complications of P' near 142°; and he attributed his steep gradient to peculiarities of Japanese crustal structure to a degree which on present evidence is most unlikely.

In Figures 6 and 7, the relevant parts of the J.-B. travel-time curves for *PKIKP*, *PKP* and PKP_2 are drawn for the case of a focal depth given by $h = 0{\cdot}095$. In Figure 6, Morita's arrival-times are shown, with his epicentral distances unaltered, relative to an origin-time of 14h 52m 43s. (This origin-time gives the best fit with the curves in the figure, and is 1s earlier than the origin-time assumed by Morita.) Morita also measured the P' amplitudes; the place of maximum amplitude, which he finds to be sharply defined, is indicated in Figures 6 and 7. Inspection of Figure 6

shows four readings which would in fact appear to correspond well to diffracted *PKP* waves, so that, at first sight, some useful support would appear to be given to the results of Section 2.

Unfortunately, however, it appears that a substantial change of epicentre may be needed. Morita adopted the provisional J.S.A. epicentre of 8°·3 S, 70°·6 W; the I.S.S., on the other hand, gives 11°·0 S, 69°·5 W, which is 2°·7 more distant from Japan; Gutenberg and Richter (1949) give 10°·9 S, 69°·5 W. The focal depth

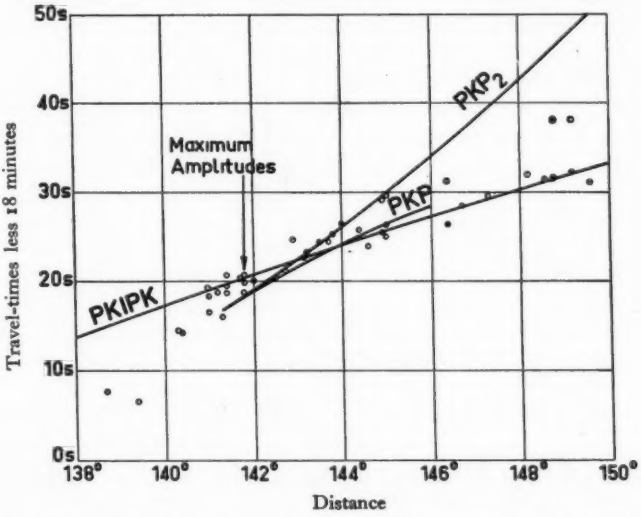

FIG. 6.—*Morita's readings at Japanese stations of P′ phases from the Upper Amazon earthquake of* 1933 *August* 29. *The assumed epicentre and origin-time are* 8°·3 S, 70°·6 W *and* 14h 52m 43s. *The curves are the J.-B. curves for a focal depth given by* $h = 0{\cdot}095$.

is given correspondingly as 640 km, $h = 0{\cdot}085$ (the I.S.S. states, however, that the depth may exceed this), and 650 km, respectively. Increases as great as 2°·7 in the Japanese epicentral distances would require the indications of the preceding paragraph to be modified.

The epicentre and origin-time were therefore carefully re-examined, using the readings quoted in the I.S.S. and the J.-B. tables of 1940. The revision gave an epicentre of 10°·4 S, 70°·3 W, an origin-time of 14h 52m 39s, and a focal depth of $h = 0{\cdot}095$ (corresponding to 635 km). This new epicentre is 1°·0 nearer Japan than the I.S.S. epicentre, and is to be preferred since it gives improved residuals. (The new epicentre markedly reduces discrepancies between important groups of residuals at North America and European stations, but leaves some unresolved discrepancies among the Central and South American residuals; thus there is still some uncertainty in the determination. In the I.S.S., it had been suggested that the shock was multiple, but the revision did not substantiate this; in fact, with the use of the 1940 J.-B. tables, many apparent anomalies in the I.S.S. readings disappeared.)

In Figure 7, Morita's readings have been shifted as a block from their position in Figure 6 to correspond to the new epicentre and origin-time. In some respects, the fit with the J.-B. curves is not quite as good in Figure 7 as in Figure 6, and the accord

with Morita's amplitude observations is a little reduced. Thus Morita's P' data suggest an epicentre nearer Japan than the new epicentre, the latter being 1°·7 further from Japan than the J.S.A. epicentre. On the other hand, any further shift of the epicentre towards Japan would reduce the consistency of the residuals for the whole set of I.S.S. data. Morita's P' readings therefore give less conclusive support to the recording of genuine diffracted *PKP* waves than they appeared to at first sight. The most that can be definitely said is that the readings at the two

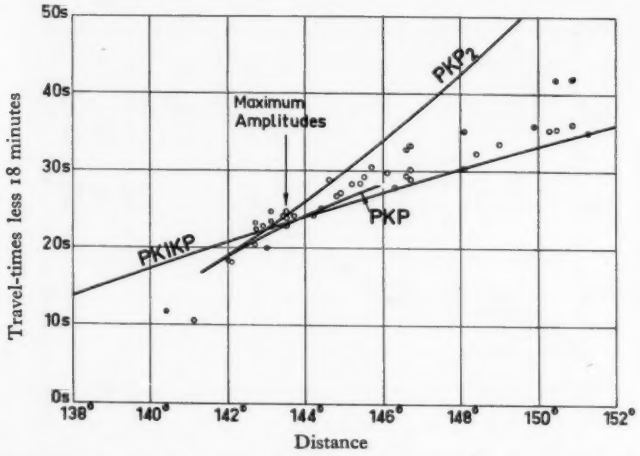

FIG. 7.—*Morita's readings of Fig.* 6 *moved so as to fit an epicentre and origin-time of* 10°·4 S, 70°·3 W *and* 14h 52m 39s. *The curves are again the J.-B. curves for* $h = 0{\cdot}095$.

Japanese stations nearest to the epicentre are clearly too early to belong to the phase *PKIKP*.

The analysis of Morita's results has been presented in a little detail in order to illustrate the difficulty of establishing the existence of diffracted *PKP* waves from purely natural earthquake data.

Other evidence of early P' readings at $\Delta < 142°$ appears in some special investigations, and suggests again that diffracted *PKP* waves have been recorded. Examples are to be found in work of Lehmann (1936, p. 28), Gutenberg & Richter (1939), and Denson (1952), although the data are too meagre for their significance to be assessed.

A survey of Riverview records over the period 1918 to 1950 revealed only seven P' readings in the range $137° < \Delta < 142°$, the distances and estimated residuals being:

139°·0, −12s (e).	140°·0, + 1s (i).
139°·2, − 3s (e).	141°·3, 0s (i).
139°·4, + 2s (i).	141°·3, −12s (e).
140°·0, + 1s (i).	

The two seismograms giving −12s were closely re-examined, and the residuals confirmed. These seismograms were interesting in that, like the South African records of the hydrogen bombs, they also showed increased amplitudes at times some 12s later than the first movements. These later movements would agree with *PKIKP*. But again, because of uncertainties in epicentres and origin-times, the data could only be regarded as suggestive.

It may be possible to show the existence of authentic observations of diffracted *PKP* waves from a study of further I.S.S. data, but the analysis of Morita's data shows how difficult it is to find cases which give the desired precision. It is hoped to present a detailed study in a later paper. In the meantime, the inconclusiveness of our results to date from natural earthquake data highlights the potential value of controlled large explosions to research on the Earth's deep interior.

4. Diffracted PKP waves and the inner core

The observations recorded in Section 2 supply a link in the chain of evidence for the existence of the inner core, in addition to indicating specific travel-times in front of the *PKP* cusp. Prior to Lehmann's work (1936), the branch now known as *PKIKP* had been assumed to be due to diffraction associated with a caustic that had been variously estimated to occur at distances from 137° to 144°. This branch having been otherwise explained by Lehmann, the question arises as to whether authentic diffracted *PKP* waves do exist for $\Delta < 142°$. The calculation of Jeffreys (1939) shows that observations of such waves are to be expected.

The present paper indicates fairly definitely that the diffracted waves have been observed in the hydrogen bomb explosions, and at times significantly earlier than the *PKIKP* times. Thus new evidence is provided that the *PKIKP* waves are indeed unconnected with diffraction.

The total evidence for the existence of the inner core now in fact consists of:

(i) Lehmann's satisfactory interpretation of the *P′* branch receding from 142° to 110°;

(ii) the supporting evidence on travel-times obtained by Gutenberg and Richter (1938), and innate in the J.-B. tables;

(iii) the amplitude test for diffracted waves carried out by Jeffreys;

(iv) the evidence of the present paper that diffracted *PKP* waves occur at times preceding *PKIKP* onsets.

The authors wish to express their gratitude to Professor A. L. Hales, Director of the Bernard Price Institute of Geophysical Research, for the very great trouble he took for us in re-examining the Kimberley and Pretoria records. We are also grateful to Mlle. Grandjean, Professor Hales, and Dr Hennessey for kindly allowing us to reproduce the seismograms shown in this paper.

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1957 *August* 12.

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The Calibration of Gravity Meters by Comparison with Pendulums

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Summary

The least squares adjustment of a set of pendulum and gravity meter observations is discussed. A general formula is obtained for the dependence of the gravity meter calibration factor on the relative weights assigned to the pendulum and gravity meter observations. Further expressions for the factor are derived when the pendulum and gravity meter networks are identical and when they contain no redundant observations. Only in the latter case is the factor independent of the relative weights of the pendulum and gravity meter observations and it is suggested that, since the relative weights may be poorly determined, observations should conform to this condition. Methods of dealing with non-independent observations are considered and the application of the results to the calibration of gravity meters is discussed.

1. Introduction

It is now generally accepted that the only satisfactory way of calibrating a gravity meter for a large interval of gravity is to compare its readings with pendulum observations over the whole of that interval. At the same time, the random observational errors of gravity meters are so much less than those of pendulums while gravity meters are so much quicker to use, that it is becoming accepted that pendulum observations should be restricted to observations over calibration ranges consisting of stations spaced more or less regularly along a north-south line, the difference of gravity between the terminal stations being equal to the interval, perhaps 3000 mgal, for which the gravity meters are to be calibrated. To that end, the differences of gravity between such sets of stations have been measured in both North America and Europe. Various methods have been used in calculating the calibration factors of gravity meters from the observations but how best to do so has not been discussed explicitly. Some consideration of this as a problem in statistics and in the structure of sets of observations is desirable: both because the relative accuracy of the pendulum observations is not high and because the observations entail spending a great deal of time and money, it is essential to see that the data are reduced in the most efficient way to obtain results of the greatest precision. The uncertainties of the calculated values should also be carefully estimated, for otherwise it may be found that quite incorrect conclusions are drawn when two gravity

meters are compared. The purpose of this paper is to discuss some of the points which arise in calculating a calibration factor, in particular the dependence of the calculated factor on the structure of the observations, and to suggest some considerations to be borne in mind when planning the observations.

In the discussion it will, for the most part, be supposed that the gravity meter reading is linearly dependent on the value of gravity and if it were known that this were strictly true it might not be necessary to make pendulum and gravity meter observations at more than two places, provided that the difference of gravity between them were large enough. In fact, it is never known that the calibration function is linear and accordingly observations must be made at a sufficient number of intermediate stations to enable any non-linearity to be detected and estimated; this is the fundamental reason, apart from considerations of the practical execution of the observations, why the stations are established with differences of about 300 mgal between successive ones and about 3000 mgal between the terminal ones. As indicated later (p. 22) it is not difficult to extend the analysis of this paper to a gravity meter with a non-linear calibration.

2. The application of the method of least squares

We consider a set of pendulum and gravity meter observations from which a set of parameters, namely the values of gravity at certain stations together with the calibration factor of the gravity meter, have to be estimated. For convenience in discussion it will be supposed, as just mentioned, that the calibration is linear so that one may speak of a "calibration factor". The principle on which the analysis of this paper depends is that the parameters shall be chosen so that the likelihood* of the observations is a maximum. We must consider the application of this principle rather carefully. Let g_i denote the values of gravity at the stations. Suppose also that the gravity meter readings are converted to gravity values using a trial value of the calibration factor which should be multiplied by a factor $(1+k)$. We have to estimate the parameters g_i and k from the observations.

Let $P(g_i k|\theta_p\theta_g H)$ denote the probability of specified values of g_i and k given the pendulum observations, denoted by θ_p, the gravity meter observations, θ_g and other relevant information H. Application of the product rule for probabilities leads to the formula

$$P(g_i k|\theta_p\theta_g H) = \frac{1}{P(\theta_p\theta_g|H)} . P(g_i k|H)P(\theta_p\theta_g|g_i kH).$$

The term $P(\theta_p\theta_g|H)$ is a constant for any given set of observations since it does not involve the parameters (g_i,k) while the prior probability of the parameters, $P(g_i k|H)$, varies slowly compared with the likelihood $P(\theta_p\theta_g|g_i kH)$. Thus the values of the parameters which have the maximum posterior probability $P(g_i k|\theta_p\theta_g H)$ are those for which the likelihood of the observations is a maximum. This argument is identical with that used by Jeffreys (1948) to establish the principle of maximum likelihood but it has been set out again in order to emphasize that we have to look for those values of the parameters which are most probable on all the data θ_p and θ_g. This, of course, is exactly what we wish to do in calculating k by comparing pendulum and gravity meter observations.

The likelihood $P(\theta_p\theta_g\,|\,g_i kH)$ may be written as the product

$$P(\theta_p|\theta_g g_i kH) . P(\theta_g|g_i kH).$$

* The likelihood of a set of observations is the probability of their occurrence given specified values of the parameters.

If we now suppose that the errors of the pendulum and gravity meter observations are independent, then the first term is equivalent to $P(\theta_p|g_i kH)$ and the joint likelihood of the pendulum and gravity meter observations is equal to the product of the likelihoods of each separately.

We suppose that errors are independent and normally distributed. Then if the residual of a typical pendulum observation of weight w_p is ϵ_p (as calculated with parameters g_i, k) and if the standard deviation of a pendulum observation of unit weight is σ_p, the likelihood of all the pendulum observations is proportional to

$$\exp\{-\tfrac{1}{2}\sigma_P^{-2}\Sigma w_p\epsilon_p^2\}.$$

With a similar notation, the likelihood of all the gravity meter observations is proportional to

$$\exp\{-\tfrac{1}{2}\sigma_G^{-2}\Sigma w_g\epsilon_g^2\}.$$

The joint likelihood of the pendulum and gravity meter observations is then proportional to

$$\exp\{-\tfrac{1}{2}\sigma_P^{-2}\Sigma w_p\epsilon_p^2-\tfrac{1}{2}\sigma_G^{-2}\Sigma w_g\epsilon_g^2\}.$$

It is worth pointing out that since the sums of weighted squares of the pendulum and gravity meter observations are divided by the variances of observations of unit weight, namely σ_G^2 or σ_P^2, the arguments of the exponentials are, as they must be, non-dimensional, and in multiplying likelihoods no difficulty arises because the pendulum and gravity meter observations are expressed in different units.

There is, nonetheless, a very real practical difficulty in formulating and minimizing the joint likelihood. It will be seen that this depends on the ratio $(\sigma_P/\sigma_G)^2$ and the difficulty lies in the estimation of this ratio, for in general σ_P and σ_G will have to be estimated from the internal discrepancies of the pendulum and gravity meter observations respectively. Now it is well known that the internal errors of groups of observations are often poor guides to the discrepancies between groups and accordingly to the weight to be given to one group relative to another. Furthermore, in the particular problem of the calibration base, the numbers of degrees of freedom on which σ_P and σ_G are determined may be rather small so that the uncertainties of each may be large. The ratio $(\sigma_P/\sigma_G)^2$ may thus be uncertain by as much as an order of magnitude and it therefore seems important to examine its influence on the calculated calibration factor and to see if the observations can be made in such a way that the calculated factor is independent of this variance ratio.

A further difficulty in minimizing the joint likelihood of the pendulum and gravity meter observations is that with general networks the inclusion of the calibration factor as an unknown to be determined greatly increases the labour of the calculations (cf. Cook 1953); a procedure in which the pendulum and gravity observations could be treated separately is therefore attractive. It will be shown that in certain simple cases this division of the calculations is possible.

With the hypotheses of normal distributions and independent observations, maximum likelihood corresponds to minimum sum of weighted squares. We establish the equations of least squares on these hypotheses.

A typical pendulum observation leads to a measured difference d_{ij} between the values of gravity at two stations i, j:

$$g_i - g_j = d_{ij}.$$

The complete set of such observation equations will be written in matrix form:

$$\mathbf{Y}_p \cdot \mathbf{g} = \mathbf{d}_p.$$

$\mathbf{Y}_p$ denotes the matrix of pendulum comparisons, $\mathbf{g}$ is the column vector with elements g_i and $\mathbf{d}_p$ is the column vector with elements d_{ij}. $\mathbf{g}$ has n elements where n is the number of stations and $\mathbf{d}_p$ has m_p elements where m_p is the number of pendulum observations. $\mathbf{Y}_p$ has m_p rows and n columns, although many of the columns may be null because many of the stations may not be occupied by pendulums.

In the simplest case all the pendulum observations are of equal weight and the non-zero elements of $\mathbf{Y}_p$ are therefore either $+1$ or -1. The analysis is general and not limited to this case.

The gravity meter equations may be written down similarly, save that we suppose that the differences of gravity have been calculated using a calibration factor to which a correction k is necessary, that is

$$g_i - g_j = d_{ij}(1+k).$$

Now the important errors of gravity meters are errors of zero and not of scale reading and are independent of the magnitude of the difference of gravity measured. We can therefore consider the random error e_{ij} as affecting the right hand side of the above equation as a whole:

$$g_i - g_j = d_{ij}(1+k) + e_{ij}.$$

This equation we may re-write as

$$g_i - g_j - kd_{ij} = d_{ij} + e_{ij},$$

and consider d_{ij} as free of error when it appears on the left hand side. In matrix form, we then have for the observation equations

$$(\mathbf{Y}_g, -\mathbf{d}_g) \cdot \begin{pmatrix} \mathbf{g} \\ k \end{pmatrix} = \mathbf{d}_g.$$

The matrix $(\mathbf{Y}_g, -\mathbf{d}_g)$ has $(n+1)$ columns and m_g rows, where m_g is the number of gravity meter observations.

The products $\mathbf{Y}_p' \mathbf{Y}_p$ and $\mathbf{Y}_g' \mathbf{Y}_g$ of the Y-matrices and their transposes which occur in the normal equations we denote by $\mathbf{N}_p$ and $\mathbf{N}_g$.

We write $(\sigma_G/\sigma_P)^2 = w$. We can now combine the pendulum and gravity meter observation equations into one matrix equation:

$$\begin{bmatrix} w^{\frac{1}{2}}\mathbf{Y}_p & \mathbf{O} \\ \mathbf{Y}_g & \mathbf{d}_g \end{bmatrix} \cdot \begin{bmatrix} \mathbf{g} \\ k \end{bmatrix} = \begin{bmatrix} w^{\frac{1}{2}}\mathbf{d}_p \\ \mathbf{d}_g \end{bmatrix}. \tag{1}$$

Normal equations formed in the usual way are then

$$\begin{bmatrix} \mathbf{N} & -\mathbf{Y}_g'\mathbf{d}_g \\ -\mathbf{d}_g'\mathbf{Y}_g & \mathbf{d}_g'\mathbf{d}_g \end{bmatrix} \cdot \begin{bmatrix} \mathbf{g} \\ k \end{bmatrix} = \begin{bmatrix} w\mathbf{Y}_p'\mathbf{d}_p + \mathbf{Y}_g'\mathbf{d}_g \\ -\mathbf{d}_g'\mathbf{d}_g \end{bmatrix}, \tag{2}$$

where $\mathbf{N} = w\mathbf{N}_p + \mathbf{N}_g$.

The solutions $\mathbf{g}$ and k of these equations give the maximum likelihood of the pendulum and gravity meter observations jointly.

A similar set of equations may be written down if it is assumed that the gravity

meter calibration includes a term proportional to the square of the value of gravity so that

$$g_i - g_j = d_{ij}(1+k) + l d_{ij}^2$$

or, in matrix form:

$$(\mathbf{Y}_g, -\mathbf{d}_g, -\mathbf{d}_g^2) \cdot \{\mathbf{g}', k, l\} = \mathbf{d}_g.$$

The braces $\{\}$ denote a column. Let $\mathbf{k}$ stand for the column $\{k, l\}$ and $\mathbf{D}_g$ for the two-column $(\mathbf{d}_g, \mathbf{d}_g^2)$; then the observational equations are

$$(\mathbf{Y}_g, -\mathbf{D}_g)\{\mathbf{g}', \mathbf{k}'\} = \mathbf{d}_g \tag{3}$$

and the normal equations are

$$\begin{bmatrix} \mathbf{N} & -\mathbf{Y}_g'\mathbf{D}_g \\ -\mathbf{D}_g'\mathbf{Y}_g & \mathbf{D}_g'\mathbf{D}_g \end{bmatrix} \cdot \begin{bmatrix} \mathbf{g} \\ \mathbf{k} \end{bmatrix} = \begin{bmatrix} w\mathbf{Y}_p'\mathbf{d}_p + \mathbf{Y}_g'\mathbf{d}_g \\ -\mathbf{D}_g'\mathbf{d}_g \end{bmatrix}. \tag{4}$$

These are formally identical with equations (2) provided the scalar k is replaced by the column $\mathbf{k}$ and the column $\mathbf{d}_g$ by the two-column $\mathbf{D}_g$. All the results of the following analysis may therefore be generalized by the same substitution.

3. The solution for k and its properties

A formal solution for k is obtained by inverting the matrix of coefficients on the left side of the normal equations (2). We call this matrix $\mathfrak{N}$. We also write

$$\theta = \mathbf{d}_g'\mathbf{d}_g - \mathbf{d}_g'\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g'\mathbf{d}_g;$$

θ is a scalar.

The elements of the inverse $\mathfrak{N}^{-1}$ are

$$\begin{aligned}
\mathfrak{N}^{11} &= \mathbf{N}^{-1} + \mathbf{N}^{-1}\mathbf{Y}_g'\mathbf{d}_g\theta^{-1}\mathbf{d}_g'\mathbf{Y}_g\mathbf{N}^{-1} && \text{(order } n_g \times n_g\text{)} \\
\mathfrak{N}^{12} &= \mathbf{N}^{-1}\mathbf{Y}_g'\mathbf{d}_g\theta^{-1} && \text{(column)} \\
\mathfrak{N}^{21} &= \theta^{-1}\mathbf{d}_g'\mathbf{Y}_g\mathbf{N}^{-1} && \text{(row)} \\
\mathfrak{N}^{22} &= \theta^{-1}.
\end{aligned}$$

The product $\mathbf{d}_g'\mathbf{d}_g$ which occurs in these and other expressions is the sum of squares of gravity meter differences.

The solution for k is

$$k = \mathfrak{N}^{21}(w\mathbf{Y}_p'\mathbf{d}_p + \mathbf{Y}_g'\mathbf{d}_g) - \mathfrak{N}^{22}\mathbf{d}_g'\mathbf{d}_g,$$

that is,

$$k = \frac{w\mathbf{d}_g'\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_p'\mathbf{d}_p + \mathbf{d}_g'(\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g' - \mathbf{I})\mathbf{d}_g}{\mathbf{d}_g'(\mathbf{I} - \mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g')\mathbf{d}_g}. \tag{5}$$

$\mathbf{I}$ denotes a unit diagonal matrix.

Further, by the usual theory of the method of least squares, we have

$$\text{variance } (k) = \mathfrak{N}^{22} \times \text{variance of observation of unit weight.}$$

Now, from the way in which w has been defined, the variance of an observation of unit weight is just σ^{-2} and so

$$\text{var } (k) = \mathfrak{N}^{22}\sigma_G^2.$$

We now investigate the properties of these formal solutions.

Certain properties of the matrices $\mathbf{N}_p$ and $\mathbf{N}_g$ will be required. In the first place, they are symmetrical. Secondly, each diagonal element is equal to the sum of the weights of all observations made at the corresponding node of the network, while any non-diagonal element is negative and numerically equal to the weight of the observation between corresponding nodes of the network.

We shall suppose, as is almost invariable practice, that all n stations are occupied with the gravity meter but only n_p ($< n$) with the pendulums. Then $\mathbf{N}_g$ is of order $n \times n$ and is positive definite but $\mathbf{N}_p$, while also formally of order $n \times n$, is of rank n_p. It may however be considered as a positive definite submatrix $\mathbf{n}_p$ of order $n_p \times n_p$ augmented by $(n-n_p)$ rows and columns of zeros. It is sometimes more convenient to consider $\mathbf{N}_p$ and sometimes $\mathbf{n}_p$.

The numerator and denominator of the expression (5) for k are both linear combinations of the elements of $w\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_p'$ and $(\mathbf{I}-\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g')$, the coefficients being products of the elements of $\mathbf{d}_g$ and $\mathbf{d}_p$. These latter elements are arbitrary for they do not depend on the structure of the observations but on the actual measured values of gravity differences, and so, if k is to be independent of w, each element of $w\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_p'$ and $(\mathbf{I}-\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g')$ must be a multiple of the same function of w, for otherwise the numerator and denominator of the expression (5) might be different functions of w that would vary with the arbitrary d's.

Thus

$$w\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_p' = f(w)\mathbf{P}$$

and

$$\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g'-\mathbf{I} = f(w)\mathbf{Q},$$

where the matrices $\mathbf{P}$, of order $m_g \times m_p$, and $\mathbf{Q}$, of order $m_g \times m_g$, are independent of w. Then remembering that $\mathbf{N} = w\mathbf{N}_p+\mathbf{N}_g$, it follows that

$$(\mathbf{PP}'+w\mathbf{QQ}')f(w) = -w\mathbf{Q}. \tag{6}$$

Each of the m_g^2 linear equations summarized in this matrix equation is of form

$$(\alpha+\beta w)f(w) = -\gamma w,$$

where α, β and γ are independent of w. Since $f(w)$ is the same for all the equations, it must be of form $-w\gamma/(\alpha+\beta w)$ and the ratios γ/α and γ/β must be the same for all the m_g^2 sets (α, β, γ).

Hence

$$\left.\begin{aligned} \mathbf{PP}' &= c_1\mathbf{Q} \\ \text{and}\quad \mathbf{Q}^2 &= c_2\mathbf{Q} \end{aligned}\right\} \tag{7}$$

where $c_1 = \alpha/\gamma$ and $c_2 = \beta/\gamma$, and where $\mathbf{Q}^2$ has been written for $\mathbf{QQ}'$ since $\mathbf{Q}$ is symmetrical.

A necessary relation between $\mathbf{N}_p$ and $\mathbf{N}_g$ is now obtained by substituting for $\mathbf{P}$ and $\mathbf{Q}$ in the first equation of set (7):

$$w(c_1+c_2w)\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{N}_p\mathbf{N}^{-1}\mathbf{Y}_g' = c_1(\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g'-\mathbf{I}).$$

$\mathbf{N}_g$ is non-singular so each side may be pre-multiplied by $\mathbf{N}_g^{-1}\mathbf{Y}_g'$ and post-multiplied by $\mathbf{Y}_g\mathbf{N}_g^{-1}$:

$$w(c_1+c_2w)\mathbf{N}^{-1}\mathbf{N}_p\mathbf{N}^{-1} = -c_1(\mathbf{N}^{-1}-\mathbf{N}_g^{-1}),$$

or, since $\mathbf{N}$ is non-singular:

$$w(c_1+c_2w)\mathbf{N}_p = -c_1(\mathbf{N}-\mathbf{N}\mathbf{N}_g^{-1}\mathbf{N}).$$

Then, on substituting for $\mathbf{N}$,

$$c_2\mathbf{N}_p = c_1\mathbf{N}_p\mathbf{N}_g^{-1}\mathbf{N}_p.$$

Since $\mathbf{N}_p$ may be singular, the general solution of this equation is

$$c_1\mathbf{N}_p = c_2(\mathbf{I}-\mathbf{X})\mathbf{N}_g$$

where $\mathbf{X}\mathbf{N}_p$ is null.

However, it will be found that although this relation between $\mathbf{N}_p$ and $\mathbf{N}_g$ is entailed by the first equation of set (7) it does not entail the equation and for this it is necessary that, in addition,

$$\mathbf{Y}_g\mathbf{N}_g^{-1}\mathbf{Y}_g' = \mathbf{I}. \tag{8}$$

The condition, $\mathbf{Y}_g\mathbf{N}_g^{-1}\mathbf{Y}_g' = \mathbf{I}$, implies that $\mathbf{Y}_g$ is of order $n \times n$ for if there are more than n rows in $\mathbf{Y}_g$, that is, more observations than unknown values of gravity, then the rows and columns of $\mathbf{Y}_g\mathbf{N}_g^{-1}\mathbf{Y}_g'$ will not all be independent and the product will be singular so that it cannot be equal to $\mathbf{I}$. Only two basic types of matrix $\mathbf{Y}_g$ satisfy this condition:

(a) The matrix $\mathbf{I}$, which represents a set of observations in which each station is compared separately with the base station.

(b) The matrix

$$\begin{bmatrix} 1 & . & . & \dots & . & . \\ -1 & 1 & . & \dots & . & . \\ . & -1 & 1 & \dots & . & . \\ \dots & \dots & \dots & \dots & . & . \\ . & . & . & . & -1 & 1 \end{bmatrix} \qquad (n \times n)$$

which represents a set of observations in which the first station is compared with the base, the second with the first, and so on, along a chain, the last station being compared with the last but one and none other.

By partitioning $\mathbf{Y}_g$ it can be seen that it may also be composed of a number of submatrices of either or both of these types.

If $\mathbf{N}_p = \mathbf{N}_g$ and $\mathbf{Y}_g\mathbf{N}_g^{-1}\mathbf{Y}_g' = \mathbf{I}$,

then $\mathbf{N} = (1+w)\mathbf{N}_g$,

$$\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{Y}_g' - \mathbf{I} = -\frac{w}{1+w}.\mathbf{I}$$

and

$$\mathbf{Y}_g\mathbf{N}^{-1}\mathbf{N}_p' = \frac{1}{1+w}.\mathbf{I}.$$

Hence

$$k = \frac{\mathbf{d}_g'(\mathbf{d}_p - \mathbf{d}_g)}{\mathbf{d}_g'\mathbf{d}_g}. \tag{9}$$

Also

$$\mathfrak{N}^{22} = \frac{1+w}{w}.\mathbf{d}_g'\mathbf{d}_g,$$

and so

$$\text{var}\,(k) = \frac{1+w}{w}.\frac{\sigma_G^2}{\mathbf{d}_g'\mathbf{d}_g},$$

or, since $\sigma_P^2 = \sigma_G^2/w$,

$$\text{var}\,(k) = (1+w)\cdot\frac{\sigma_P^2}{\mathbf{d}_g'\,\mathbf{d}_g}. \tag{10}$$

These simple results for this elementary case may of course be obtained by less elaborate algebra.

It will be noticed that although k is independent of w, its variance is not.

The expressions for k and its variance can also be put in simple forms when the pendulum and gravity meter networks, although redundant, are identical. For we may suppose each network to be adjusted separately giving sets of residuals $\boldsymbol{\epsilon}_p$, $\boldsymbol{\epsilon}_g$, which are related to the observations by the equations

$$\mathbf{d}_p - \boldsymbol{\epsilon}_p = \mathbf{Y}_p \mathbf{N}_p^{-1} \mathbf{Y}_p'\,\mathbf{d}_p$$

and

$$\mathbf{d}_g - \boldsymbol{\epsilon}_g = \mathbf{Y}_g \mathbf{N}_g^{-1} \mathbf{Y}_g' \mathbf{d}_g.$$

We also have that $\mathbf{Y}_p = \mathbf{Y}_g$ and $\mathbf{N}_p = \mathbf{N}_g = \mathbf{N}(1+w)^{-1}$. Then

$$\theta = \frac{w}{1+w}\mathbf{d}_g'\,\mathbf{d}_g + \frac{1}{1+w}\mathbf{d}_g\,\boldsymbol{\epsilon}_g$$

and

$$k = \frac{\mathbf{d}_g'\,(\mathbf{d}_p - \boldsymbol{\epsilon}_p - \mathbf{d}_g + \boldsymbol{\epsilon}_g) - w^{-1}(1+w)\mathbf{d}_g'\,\boldsymbol{\epsilon}_g}{\mathbf{d}_g'\,\mathbf{d}_g + w^{-1}\,\mathbf{d}_g'\,\boldsymbol{\epsilon}_g} \tag{11}$$

while

$$\text{var}\,(k) = (1+w)\frac{\sigma_P^2}{\mathbf{d}_g'\,\mathbf{d}_g}\left[1 + \frac{\mathbf{d}_g'\,\boldsymbol{\epsilon}_g}{w\mathbf{d}_g'\,\mathbf{d}_g}\right]^{-1}. \tag{12}$$

It follows that if k is to be independent of w, the residuals $\boldsymbol{\epsilon}_p$ and $\boldsymbol{\epsilon}_g$ must be all zero, that is, in general, there must be no redundant observations, the condition already established.

These formulae should provide convenient means of calculating k and its variance once the pendulum and gravity meter observations have been adjusted independently. In any well observed networks, the values of the residuals $\boldsymbol{\epsilon}_p$ or $\boldsymbol{\epsilon}_g$ will be very small compared with the differences $\mathbf{d}_p$ and $\mathbf{d}_g$ and it may be possible to ignore the terms containing them and, in consequence, the dependence of k on w.

Simple exact solutions for k and its variance are thus available when the pendulum and gravity meter network are identical; unfortunately it does not seem possible to obtain any useful general results when the two networks are not identical.

The condition that there should be no redundant observations does not mean that the gravity meter observations may not be made as subsidiary networks between two points of the calibration network. Thus, suppose that the differences $(i-j)$, $(j-k)$ between stations i, j, k have been measured with pendulums. If a network of gravity meter observations is now established between i and j and adjusted, a calculated value is obtained for the gravity meter difference between i and j. But if this subsidiary network includes just the two points, i and j, of the pendulum network and none other, then the calculated difference $(i-j)$ has an uncertainty independent of that of gravity meter observations between any other pair of points of the pendulum network and may be assigned a weight determined from the standard deviation calculated in the adjustment of the subsidiary network.

We may generalize the formal solutions to obtain the co-variance of the estimates of the calibration factors of two gravity meters calibrated against the same pendulum observations. With an obvious extension of the above notation we have the following observation equations:

$$\begin{bmatrix} w^{\frac{1}{2}}\mathbf{Y}_p & \mathbf{O} & \mathbf{O} \\ \mathbf{Y}_{g1} & -\mathbf{d}_{g1} & \mathbf{O} \\ \mathbf{Y}_{g2} & \mathbf{O} & -\mathbf{d}_{g2} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{g} \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} w^{\frac{1}{2}}\mathbf{d}_p \\ \mathbf{d}_{g1} \\ \mathbf{d}_{g2} \end{bmatrix}.$$

These lead to the following matrix $\mathfrak{N}$ for the normal equations:

$$\begin{bmatrix} \mathbf{N} & \mathbf{Y}'\mathbf{d} \\ \mathbf{d}'\mathbf{Y} & \mathbf{D} \end{bmatrix}.$$

Here

$$\begin{aligned} \mathbf{N} &= w\mathbf{N}_p + \mathbf{N}_{g1} + \mathbf{N}_{g2}, \\ \mathbf{Y}'\mathbf{d} &= -\mathbf{Y}_1\mathbf{d}'_{g1} - \mathbf{Y}'_{g2}\mathbf{d}_{g2}, \\ \mathbf{d}'\mathbf{Y} &= -\mathbf{d}'_{g1}\mathbf{Y}_{g1} - \mathbf{d}'_{g2}\mathbf{Y}_{g2}, \end{aligned}$$

and

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}'_{g1}\mathbf{d}_{g1} & \mathbf{O} \\ \mathbf{O} & \mathbf{d}'_{g2}\mathbf{d}_{g2} \end{bmatrix}.$$

Proceeding as before with the inversion of $\mathfrak{N}$ we find that $\mathfrak{N}^{22}$, now a 2×2 matrix, is equal to the inverse of

$$\begin{bmatrix} \mathbf{d}'_{g1}\mathbf{d}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{d}_{g2}\mathbf{d}_{g2} \end{bmatrix} - \begin{bmatrix} \mathbf{d}'_{g1}\mathbf{Y}_{g1} \\ \mathbf{d}'_{g2}\mathbf{Y}_{g2} \end{bmatrix} \cdot \mathbf{N}^{-1} \cdot (\mathbf{Y}'_{g1}\mathbf{d}_{g1},\ \mathbf{Y}'_{g2}\mathbf{d}_{g2}).$$

Assuming that all three networks are identical, we find that

$$(\mathfrak{N}^{22})^{-1} = \begin{bmatrix} \mathbf{d}'_{g1}\mathbf{d}_{g1} & \mathbf{O} \\ \mathbf{O} & \mathbf{d}'_{g2}\mathbf{d}_{g2} \end{bmatrix} - \frac{1}{2+w} \begin{bmatrix} \mathbf{d}'_{g1}(\mathbf{d}_{g1} - \boldsymbol{\epsilon}_{g1}) & \mathbf{d}'_{g1}(\mathbf{d}_{g2} - \boldsymbol{\epsilon}_{g2}) \\ \mathbf{d}'_{g2}(\mathbf{d}_{g1} - \boldsymbol{\epsilon}_{g1}) & \mathbf{d}'_{g2}(\mathbf{d}_{g2} - \boldsymbol{\epsilon}_{g2}) \end{bmatrix}.$$

Further assuming that the residuals, $\boldsymbol{\epsilon}$, may be ignored, either because they are relatively small or because they are identically zero,

$$(\mathfrak{N}^{22})^{-1} = \frac{1}{2+w} \begin{bmatrix} (1+w)S_{11} & -S_{12} \\ -S_{12} & (1+w)S_{22} \end{bmatrix},$$

where $S_{11} = \mathbf{d}'_{g1}\mathbf{d}_{g1}$, and so on.

The co-variance of k_1 and k_2 is equal to σ_G^2 multiplied by the off-diagonal element of $\mathfrak{N}^{22}$, that is,

$$\operatorname{cov}(k_1 k_2) = \frac{(2+w)\,S_{12}}{(1+w)^2\,S_{11}\,S_{22} - S_{12}^2}. \tag{13}$$

Since the elements of $\mathbf{d}_{g1}$ will be very nearly equal to the corresponding elements of $\mathbf{d}_{g2}$, we have the further approximations $S_{11} = S_{22} = S_{12}$, and

$$\operatorname{cov}(k_1 k_2) = \frac{1}{w} \cdot \frac{\sigma_G^2}{S_{11}}.$$

Thus for this simple case,

$$\mathrm{cov}\,(k_1 k_2) = \frac{1}{1+w}\,.\,\mathrm{var}\,(k).$$

It follows that

$$\mathrm{var}\,(k_1 - k_2) = \frac{w}{1+w}\,.\,\mathrm{var}\,(k).$$

This completes the investigation of the properties of the solution for k when all observations are independent and normally distributed. A simple exact solution for k is available when the gravity meter and pendulum networks are identical.

4. Effects of correlation between observations

The hypothesis of uncorrelated observations, on which the analysis of Section 3 depends, is often not satisfied in practice. If, for instance, a gravity meter is read at three places, A, B and C, the independent observations are the three readings and the derived differences of gravity are correlated through the readings common to them: if the variance of a single reading is v, the variance of a gravity difference is $2v$ and the co-variance of two differences with a common reading is v. The correlation will be more complex if there is appreciable instrumental drift.

We consider therefore how the method of least squares is extended to correlated observations and whether it is possible to obtain results similar to those of Section 3 when observations are correlated. The likelihood of a set of correlated observations with the column of residuals $\boldsymbol{\epsilon}$ is proportional to $(-\frac{1}{2}\boldsymbol{\epsilon}'\mathbf{W}\boldsymbol{\epsilon})$ where $\mathbf{W}$ is the inverse of the co-variance matrix of the observations. $\mathbf{W}$ is diagonal for uncorrelated observations and the matrix product then reduces to $\Sigma w_i \epsilon_i^2/\sigma^2$. $\mathbf{W}$ is a matrix of weights. To minimise this product with respect to the parameters we choose linear functions ξ of the original observations given by

$$\boldsymbol{\epsilon} = \mathbf{T}\boldsymbol{\xi}.$$

The matrix $\mathbf{T}$ of this transformation is chosen so that the matrix $\mathbf{W}$ is transformed into a unit diagonal:

$$\mathbf{T}'\mathbf{W}\mathbf{T} = \mathbf{I}.$$

Then

$$\boldsymbol{\epsilon}'\mathbf{W}\boldsymbol{\epsilon} = \boldsymbol{\xi}'\mathbf{T}'\mathbf{W}\mathbf{T}\boldsymbol{\xi} = \boldsymbol{\xi}'\boldsymbol{\xi},$$

so that the matrix product of the ϵ's has been transformed into a sum of squares of the ξ's for which normal equations can be found in the usual way.

Naturally it is presupposed that the matrix $\mathbf{W}$ is known and the scope of this treatment is limited by the fact that $\mathbf{W}$ will often not be known because of the complexity of the correlation. When the analysis of Section 3 is repeated for correlated observations, it will be found that, to obtain results analogous to those for identical pendulum and gravity meter networks, not only do the networks have to be identical but also the W-matrices have to be the same. Circumstances in which this last condition would be fulfilled would be very artificial.

Another possible approach when instrumental drift can be ignored is to proceed as has been done in adjusting certain pendulum observations (Garland & Cook 1955) and to take the instrumental readings rather than the deduced differences of gravity as the data to be adjusted.

Considering, first, the gravity meter observation equations, let the sth reading at station i be r_{si}, expressed in milligals using a trial calibration factor. Then if the value of gravity is g_i,

$$g_i = a + r_{si}(1+k)$$

where a is a constant.

At station o, gravity g_0 is assumed known. Choose trial values g_i^*, a^*, and put

$$\gamma_i = g_i^* - g_i, \quad \alpha = a - a^*.$$

Then $\gamma_i - \alpha - kr_{si} = r_{si} - (g_i^* - a^*) = \rho_{si}$ say.

In general, there will be n_{gi} such gravity meter equations at each station so that, combining them,

$$n_{gi}\gamma_i - n_{gi}\alpha - k\sum_s r_{si} = \sum_s \rho_{si}.$$

At the origin, γ_i is zero and

$$-n_{g0}\alpha - k\sum_s r_{s0} = \sum_s \rho_{s0}.$$

These equations may be written in matrix form:

$$(\mathbf{D}_g, -\mathbf{n}_g, -\mathbf{r}_g)\cdot\{\gamma, \alpha, k\} = \boldsymbol{\rho}_g.$$

In this, $\mathbf{D}_g$ is diagonal with elements n_i (excluding n_0); $\mathbf{n}_g$ is the column with elements $-n_{gi}$, $-n_{g0}$; $\mathbf{r}_g$ is the column with elements $\sum_s r_{si}$ and $\boldsymbol{\rho}_g$ is that with elements $\sum_s \rho_{si}$, i in each case including o.

The derivation of the pendulum equations is less straightforward. At each station, the half-period, S_i, is given by

$$S_i^2 = \pi^2 L/g_i$$

where L is the length of the equivalent simple pendulum.

Let $\pi^2 L = l$ and take trial values g_i^*, l^*, with γ_i and λ as differences from trial values. Also, let

$$S_i^{*2} = l^*/g_i^*.$$

Then

$$\frac{\gamma_i}{g_i^*} - \frac{\lambda}{l^*} = 1 - \frac{S_i^2}{S_i^{*2}} = s_i, \text{ say,}$$

with

$$-\frac{\lambda}{l^*} = s_0.$$

Now the difference between g_0^* and g_i^* is certainly less than $5\times10^{-3}\,g_0$ and if l^* is properly chosen λ will be of the same order as $l^* s_i$, each of these quantities being of the order of the standard deviation of a single observation. The error committed in replacing $\lambda g_i^*/l^*$ by $\lambda g_0^*/l^*$ will therefore be at most $5\times10^{-3} g_0^* s_i$ compared with the standard deviation of a single observation of order $g_0^* s_i$.

Accordingly the pendulum equations may be taken to be:

$$n_{pi}\gamma_i - n_{si}\lambda g_0^*/l^* = \sum_s g_i^* s_{si}$$

and

$$-n_{s0}\lambda g_0^*/l^* = \sum_s g_0^* s_{s0},$$

or, in matrix form,

$$(\mathbf{D}_p, -\mathbf{n}_p)\{\gamma, \lambda g_0^*/l^*\} = \boldsymbol{\rho}_p \quad .$$

where $\boldsymbol{\rho}_p$ is the column of elements, $\Sigma g_i^* s_{si}$, $\Sigma g^* s_{s0}$.

Since the parameter α does not enter the pendulum equations, nor the parameter $\lambda g_0^*/l$ the gravity meter equations, the matrices $\mathbf{N}_p$ and $\mathbf{N}_g$ which are derived from these equations cannot be identical even if the number of pendulum and gravity meter observations at each station are the same, and so k is never independent of w unless the normal equations for α and λ are independent of the others.

It appears, then, that if the observed gravity differences are correlated, there is no extension of the analysis of Section 3 that will give a simple result for k and its variance. It follows that if an exact solution for k with a well-established variance is required, then the observations should be made in such a way that the derived differences of gravity are independent.

It is often possible to argue that if correlated observations are adjusted as if they were independent, then the expectations of the estimates of the parameters are the same as would be obtained from a correct adjustment taking account of the correlation, since, in each case, the calculated parameters depend linearly on the experimental errors and the expected values of such linear combinations are zero. The variances of the parameters depend of course on the coefficients of these linear combinations. This argument does not apply to the solution for k since it will be seen that k itself includes sums of squares of errors, the expected values of which are not zero.

5. Other procedures

Martin (1955) has compared pendulum and gravity meter differences between numerous pairs of stations and has then summed the respective differences to give sums of intervals ΣP and ΣG. He then puts k equal to $(\Sigma P - \Sigma G)/\Sigma G$. This is evidently not the most probable value.

Morelli (1951) has used a similar method but, by proceeding by least squares, has effectively put k equal to

$$\Sigma G(\Sigma P - \Sigma G)/\Sigma G^2.$$

This is the same as formula (9) above provided there are no closed loops of observations.

Kneissl (1956) adopts a quite different method. He first adjusts the pendulum and gravity meter networks separately and then, choosing one station as origin, he compares the adjusted pendulum and gravity meter differences between this and all other stations. Treating these as observations equations, he obtains the correction to the gravity meter calibration factor by least squares, arriving at the formula

$$k = \frac{\mathbf{g}_g'(\mathbf{g}_p - \mathbf{g}_g)}{\mathbf{g}_g'\,\mathbf{g}_g},$$

where $\mathbf{g}_p$ and $\mathbf{g}_g$ are the columns of adjusted gravity values, the values at the origin being zero. In the notation of Section 3,

$$\mathbf{g}_p = \mathbf{N}_p^{-1}\mathbf{Y}_p'\,\mathbf{d}_p,$$

$$\mathbf{g}_g = \mathbf{N}_g^{-1}\mathbf{Y}_g'\,\mathbf{d}_g.$$

If the two networks are identical, the expression for k is then

$$\frac{\mathbf{d}_g' \mathbf{Y}_g \mathbf{N}_g^{-2} \mathbf{Y}_g' (\mathbf{d}_p - \mathbf{d}_g)}{\mathbf{d}_g' \mathbf{Y}_g \mathbf{N}_g^{-2} \mathbf{Y}_g' \mathbf{d}_g}.$$

The corresponding expression as derived in Section 3 is formula (11).

The reason for the different result given by Kneissl's method is that the adjustment of the $\mathbf{g}_p$ and $\mathbf{g}_g$ is carried out as if the elements of each were uncorrelated whereas in fact they are far from independent. In addition, the different weights of the pendulum and gravity meter observations are ignored.

It is possible to write down the correlation matrices of the g_p and g_g and then to transform these to new independent variables. When these variables are used in place of the g_p and g_g in the least-squares computation of k, the formula (11) of Section 3 is reproduced.

It is evident that it is just as straightforward to use formula (11) as to use Kneissl's formula.

6. Conclusion

In preparing a programme of observations for the calibration of gravity meters, questions other than the formal structure of the observations have to be considered, and the expense and practical convenience of a scheme may have to be balanced with theoretical requirements. Because it is costly and time-consuming to carry instruments from place to place over long distances, observers are anxious to occupy the stations with as few journeys as possible and may consider that the travelling needed to measure each difference independently is excessive.

The most important theoretical requirement is that the pendulum and gravity meter networks should be identical. It is also very desirable that all the observed differences of gravity should be independent, in particular because only then can a valid estimate readily be made of the variance of the estimate of k.

One way of saving time and money is to cut out redundant observations from the networks. The objects of redundant observations are to increase the precision of the results by increasing the number of degrees of freedom and to reveal gross errors. For neither of these purposes are redundant observations very efficient in the present circumstances. The precision of the estimate of k is principally determined by the overall difference of gravity provided that the distribution of stations is adequate for the estimation of a quadratic term. If there are $n+1$ stations involved, the minimum number of observations is $2n$ (n pendulum, n gravity meter) while the number of parameters to be determined is $n+1$ (n unknown gravity values and k). The number of degrees of freedom is therefore $(n-1)$. To obtain a worthwhile improvement in precision, this number should be at least doubled, involving about fifty per cent more observations than the minimum and the available effort could be better employed in increasing the overall difference of gravity. As for gross errors, these will be revealed since each difference is measured with both pendulums and gravity meters.

7. Acknowledgments

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A Study on the Direction of Arrival of Microseisms at Kew Observatory

H. M. Iyer

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Summary

A modification of Darbyshire's correlation method to study the direction of arrival of microseisms is used to track three fast-moving cyclonic depressions in the Atlantic during the periods 1951 October, 1952 October and 1956 December. Good agreement between the calculated and actual bearings of the storm centre is obtained when the depression is reasonably far away from coastal regions and has a simple meteorological situation. When the storm is close to the coast, however, multiple sources of microseisms make tracking unreliable. Filtering the waves and using a narrow band about the spectral peak and using longer records improves the accuracy. In all the cases studied, the microseisms contain an appreciable amount of Love waves. The ratio of intensity of Love waves to Rayleigh waves seems to depend on the position and intensity of the storm centre.

1. Introduction

There is strong evidence to show that cyclonic storms and associated sea waves in the Atlantic are the primary cause of microseisms of period range 3 to 10 seconds recorded in Great Britain (Darbyshire 1950, 1954, Deacon 1947, Geddes 1957, Iyer 1956) and the theory of Longuet-Higgins (1950) accounts for most of the observed features. It has been found that almost all cyclonic storms in the Atlantic within range of the seismograph station at Kew Observatory give rise to microseims of noticeable amplitude in the seismograph records there. But for using the information from these records for successfully tracking storms, a reliable direction-finding technique is quite essential. Much work has been done in this field by various workers all over the world, and new methods have been put forward from time to time. The chief among them are:

(1) Tripartite method (Krug 1937, Macelwane 1946).
(2) Lee's (1935) method.
(3) Bath's (1952) parabola method.
(4) Gilmore's (1952) microratio technique.
(5) Darbyshire's (1954) correlation method.

All of these methods have met with varying amounts of success. Of these the last mentioned seems to be quite a promising approach to a complicated problem. It is now accepted by most workers that microseisms are generally a combination of Rayleigh and Love waves, the vertical component seismograph recording only

Rayleigh waves and the two horizontal components a mixture of the two. The correlation method aims at separating the waves by measuring the cross correlation coefficients between the various components. The necessary information for estimating the direction of arrival of the waves and the Love/Rayleigh wave ratio is obtained from a single three-component station, and two such stations ought to pin-point the source of microseisms which is the storm region. The present investigation is mainly aimed at modifying Darbyshire's method so that it becomes more universally applicable for the study of the composition and direction of arrival of microseismic waves.

2. Theory

Throughout this analysis it will be assumed that the microseisms are arriving from a single direction.

For Rayleigh and Love waves arriving at the recording station with amplitudes R and L respectively making an angle θ with the North-South direction, the r.m.s. values of the three components can be written as

EW:

$$\bar{x} = (\overline{R^2}\sin^2\theta + \overline{L^2}\cos^2\theta)^{\frac{1}{2}}, \tag{1}$$

NS:

$$\bar{y} = (\overline{R^2}\cos^2\theta + \overline{L^2}\sin^2\theta)^{\frac{1}{2}}, \tag{2}$$

Vertical:

$$\bar{z} = \frac{1}{k}\bar{R}, \tag{3}$$

where k is the Rayleigh wave constant, i.e. the ratio of the horizontal to vertical amplitude of Rayleigh waves.

The maximum cross-correlation coefficients between the three components can be written as

EW and Vertical:

$$r_{xz} = \overline{R}\sin\theta(\overline{R^2}\sin^2\theta + \overline{L^2}\cos^2\theta)^{-\frac{1}{2}} = \overline{R}\sin\theta/\bar{x}. \tag{4}$$

NS and Vertical:

$$r_{yz} = \overline{R}\cos\theta(\overline{R^2}\cos^2\theta + \overline{L^2}\sin^2\theta)^{-\frac{1}{2}} = \overline{R}\cos\theta/\bar{y}. \tag{5}$$

EW and NS:

$$r_{xy} = (\overline{R^2}+\overline{L^2})\sin\theta\cos\theta(\overline{R^2}\sin^2\theta + \overline{L^2}\cos^2\theta)^{-\frac{1}{2}}(\overline{R^2}\cos^2\theta + \overline{L^2}\sin^2\theta)^{-\frac{1}{2}}$$

$$= (\overline{R^2}+\overline{L^2})\sin\theta\,.\,\cos\theta/\bar{x}\bar{y}$$

$$= r_{xz}r_{yz} + r_L. \tag{6}$$

where r_L is the correlation coefficient between the Love waves in the horizontal components.

Darbyshire (1954) has used (4), (5) and (6) and obtained equations which can be solved for values of θ and L/R using the values of the three correlation coefficients.

The author attempted to use these equations for continuously tracking a storm

of February 1949 and found that r_{xy} was a very small quantity, generally of the order of the product between r_{xz} and r_{yz} (Table 1)

Table 1

Comparison of correlation coefficients: storm of 1949 February

Date	Time	$r_{xz} \cdot r_{yz}$	r_{xy}	Date	Time	$r_{xz} \cdot r_{yz}$	r_{xy}
6	1200	0·20	0·25	7	1200	0·04	0·23
	1500	0·16	0·10		1500	0·04	0·26
	1800	0·08	0·30		1800	0·23	0·48
	2100	0·11	0·02		2100	0·10	0·21
7	0000	0·13	0·10	8	0000	0·24	0·40
	0300	0·10	0·09		0300	0·33	0·25
	0600	0·09	0·15		0600	0·52	0·58
	0900	0·06	0·27		0900	0·37	0·27

In many other cases also r_{xy} was found to be generally small. The contribution r_L in equation (6) is therefore very small and one is led to conclude either that the proportion of Love waves present is very small or that the Love wave components are not very highly correlated. As, however, the ratio of the two horizontal components remains more or less constant, even though the storm veers through 90° (Table 2), the Love wave contribution must be appreciable and the second alternative is more likely.

Table 2

Ratio of the two horizontal components: storm of 1949 February

Date	Time	x/y	Date	Time	x/y
6	1200	1·83	7	1200	1·53
	1500	1·40		1500	1·69
	1800	1·70		1800	1·61
	2100	1·31		2100	2·05
7	0000	1·18	8	0000	2·68
	0300	1·42		0300	1·52
	0600	1·17		0600	1·58
	0900	1·85			

It is clear, however, that in the circumstances r_{xy} cannot be a very reliable quantity for calculating the direction of arrival of microseisms.

So equations (4) and (5) were solved to give new formulae for θ and L/R:

$$\tan\theta = \frac{r_{xz}}{r_{yz}} \cdot \frac{\bar{x}}{\bar{y}}, \tag{7}$$

$$(1 + \overline{L^2}/\overline{R^2}) = \frac{\bar{x}^2 + \bar{y}^2}{r_{xz}^2 \cdot \bar{x}^2 + r_{yz}^2 \cdot \bar{y}^2}. \tag{8}$$

Equations (7) and (8) were used in a preliminary study of the 1949 storm (Iyer, 1956). But it is now considered better not to use these equations as absolute intensities of horizontal components are involved. Unfortunately, microseismic data are often obtained from long period earthquake seismographs and it is not

advisable to use their calibration in the microseism range. Hence new equations involving only r_{xz} and r_{yz} are derived for θ and L/R.

Equations (4) and (5) can easily be transformed and re-written in the form:

$$(r_{xz}^{-2}-1)^{\frac{1}{2}} = \overline{L}\cot\theta/\overline{R}, \tag{9}$$

$$(r_{yz}^{-2}-1)^{\frac{1}{2}} = \overline{L}\tan\theta/\overline{R}, \tag{10}$$

and hence

$$\tan^2\theta = \frac{(r_{yz}^{-2}-1)^{\frac{1}{2}}}{(r_{xz}^{-2}-1)^{\frac{1}{2}}}, \tag{11}$$

$$\overline{L^2}/\overline{R^2} = (r_{yz}^{-2}-1)^{\frac{1}{2}}(r_{xz}^{-2}-1)^{\frac{1}{2}}. \tag{12}$$

Equations (11) and (12) are quite neat in the sense that they involve the phase relationship between the Rayleigh waves only. The correlation coefficients between the horizontal and vertical components are quite significant and can be measured accurately.

3. Data used and method of analysis

Three storms are chosen for this study:

1. 1951 October 8 and 9
2. 1952 October 27, 28 and 29
3. 1956 December 9, 10 and 11.

The tracks of the storms and allied synoptic situations are shown in Figures 1, 2 and 3. The seismograph records used were kindly lent by the authorities of Kew Observatory. The records are magnified and converted into black and white profile form suitable for analysis on photoelectric instruments.

The correlation coefficients were measured on the correlation meter first described by Tucker (1952) and later used by Darbyshire (1954). The instrument has since been modified in two ways. The two scanning lines of light are also made to act on a photomultiplier device which gives a trace on an oscilloscope so that the coincidence of the two lines can be accurately determined. Secondly, a set of five narrow band pass filters have been incorporated so that the correlation coefficients between waves over a range of 1s period, from 5s to 10s, can be determined.

The frequency spectra and intensities were measured on the N.I.O. Frequency Analyser (Tucker 1956).

This instrument records the amplitude of the Fourier components of the input wave form. Using this, the amplitude spectrum for the vertical component of microseisms has been constructed by squaring and adding the individual peaks over short intervals (1s for 1951 and 1956 storms, and $\frac{1}{2}$s for the 1952 storm) and taking the square root of the total value.

In the case of the 1951 and 1952 storms, 10min samples of records were used, once in three hours during the earlier part of the storm and once in six hours later. Only total correlations (i.e. without any filtering) were used. For the 1956 storm, however, 45min records in 15min lengths were used every three hours during the earlier part and 15min records every six hours during the later part. Here the correlations were measured for the total spectrum as well as the 6–7s band.

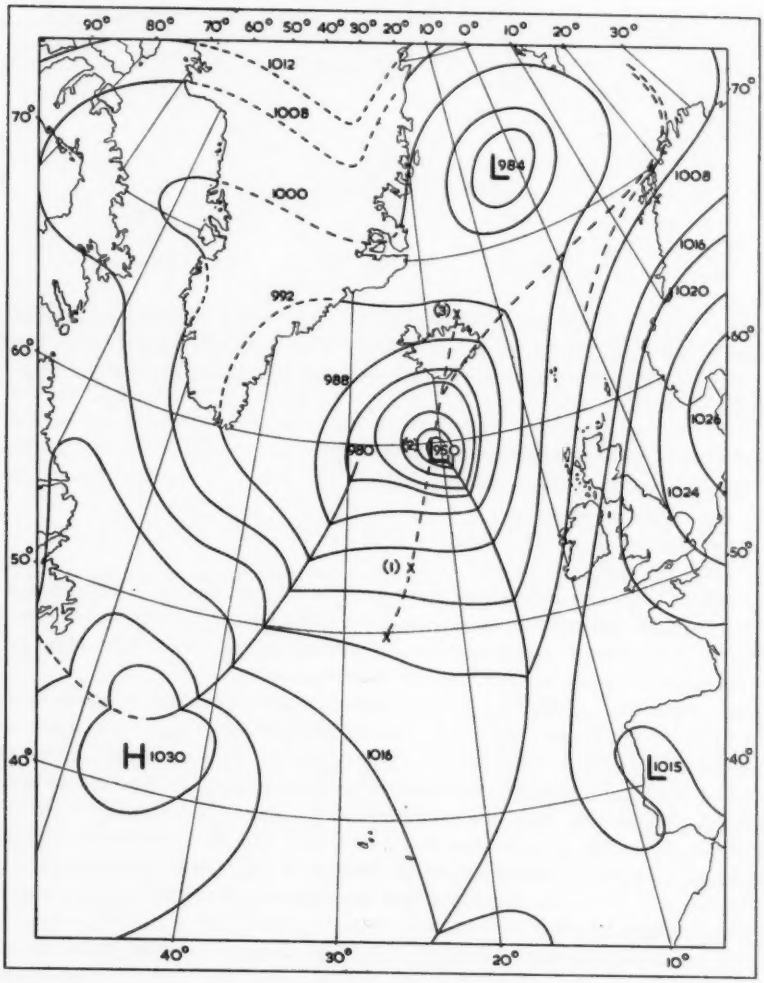

FIG. 1.—*Track of the storm which produced microseisms on* 1951 *October* 8 *and* 9. (1) 1200, *October* 8; (2) 0000, *October* 9; (3) 1200, *October* 9.

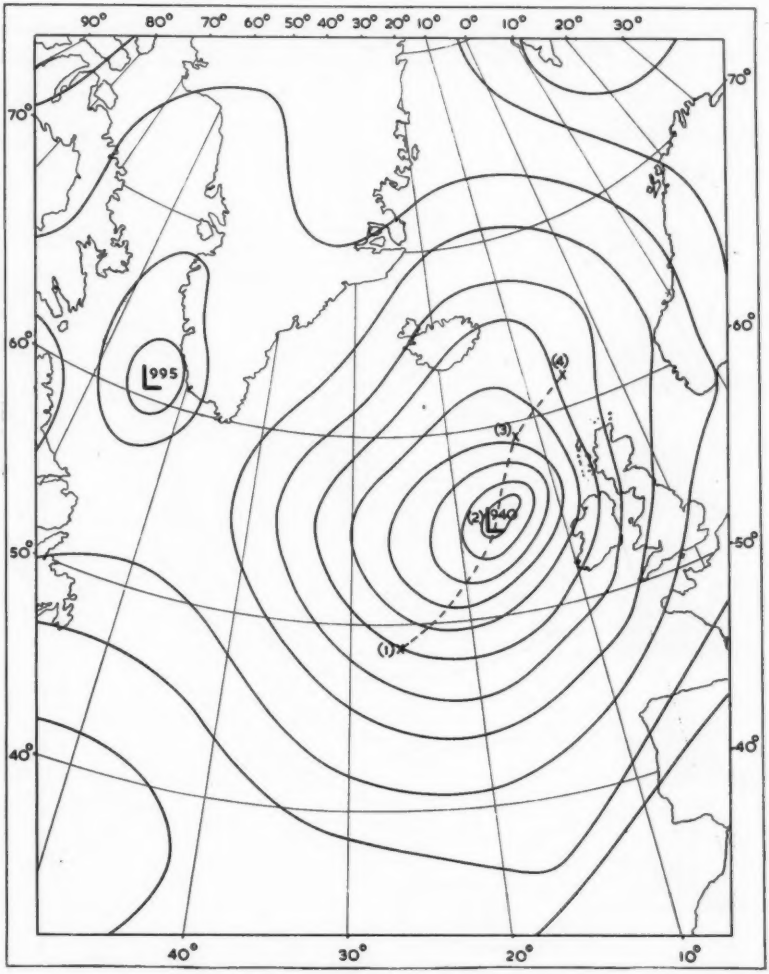

FIG. 2.—*Track of the storm which produced microseisms from* 1952 *October* 27 *to* 29. (1) 1200, *October* 27; (2) 0000, *October* 28; (3) 1200, *October* 28; (4) 0000, *October* 29.

Bearings and L/R were evaluated using equations (11) and (12).

In all three storms the phase difference between EW and vertical components was very nearly zero and that between NS and vertical either zero or 180° depend-

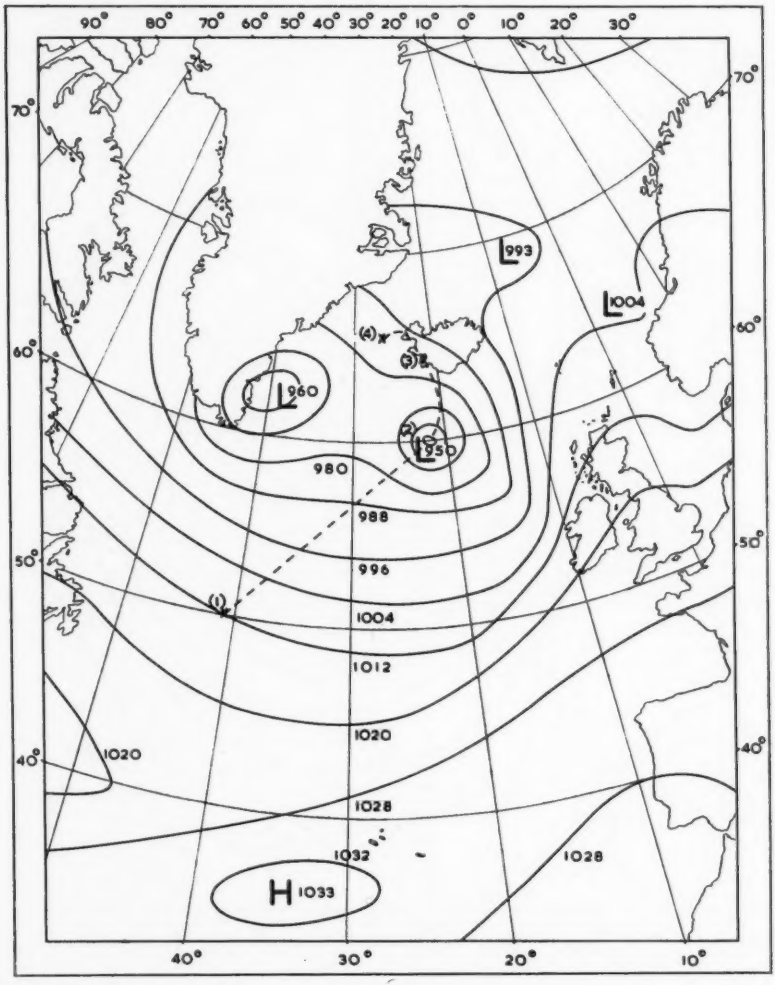

FIG. 3.—*Track of the storm which produced microseisms from* 1956 *December* 9 *to* 11. (1) 1200, *December* 9; (2) 0000, *December* 10; (3) 1200, *December* 10; (4) 0000, *December* 11.

ing on whether the bearing of the storm was between 180° and 270°, or between 270° and 360°. This gives the right relationship between the phases of the horizontal components, but a shift in the phase of the vertical component; the theoretical value of the phase difference between the vertical and horizontal component for Rayleigh waves is 90°.

4. Discussion of results

4.1 *Storm of* 1951 *October* (*Figure* 4)

The calculated bearings follow quite closely the actual movement of the storm. Except for the bearing at 0600 the agreement seems to be very good. The L/R ratio decreases as the storm moves and intensifies. It is not clear whether this effect is due to selective refraction and absorption of Rayleigh waves, the Love waves

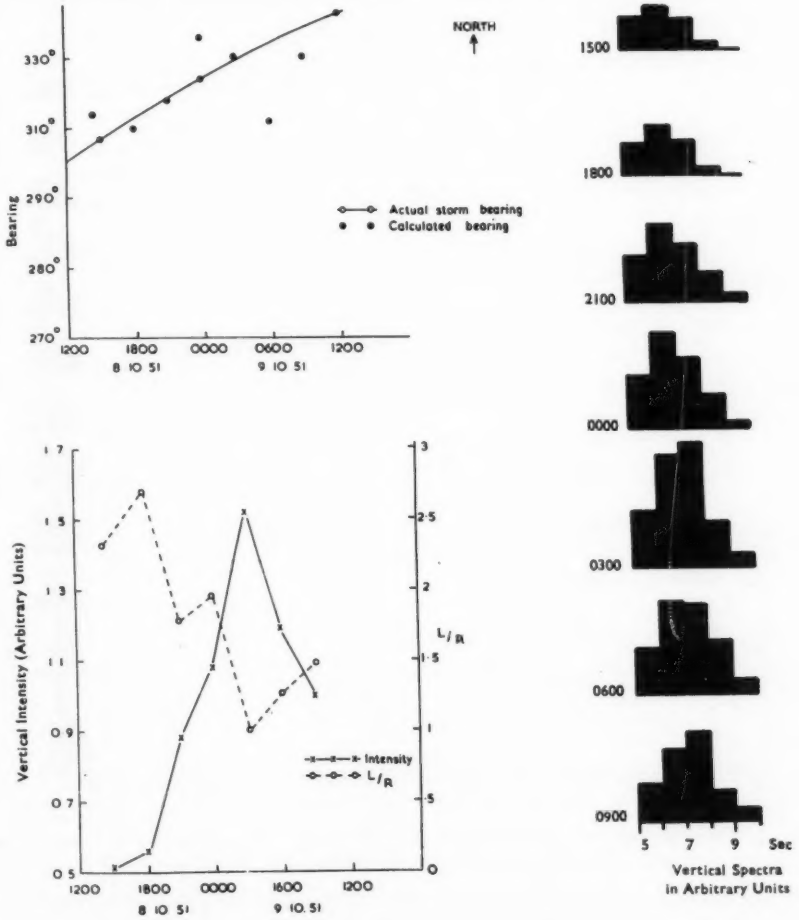

FIG. 4.—*Variation with time of direction of arrival, intensity, L/R ratio and period spectrum for microseisms generated by the storm of* 1951 *October.*

contributing a kind of noise to the total intensity of the storm. The Rayleigh wave spectrum shows a marked shift from 6–7 to 7–8s as the storm progresses. As there has been very little sea wave activity during the period under investigation (see Darbyshire 1954), this can only be attributed to the intensification of the storm. This is an indirect support to the theory of origin of microseisms by sea waves in the storm area, as the sea wave period should increase when the winds in a cyclonic

storm intensify, and microseismic periods would be half of the corresponding sea wave periods.

4.2 *Storm of* 1952 *October* (*Figure* 5)

The calculated bearings, even though quite close to the storm position in a few instances, do not show any decided tendency to follow the storm. On the other hand, one is led to infer the existence of a fixed source between 280° and 300°. This discrepancy can be explained by an examination of the storm track and the

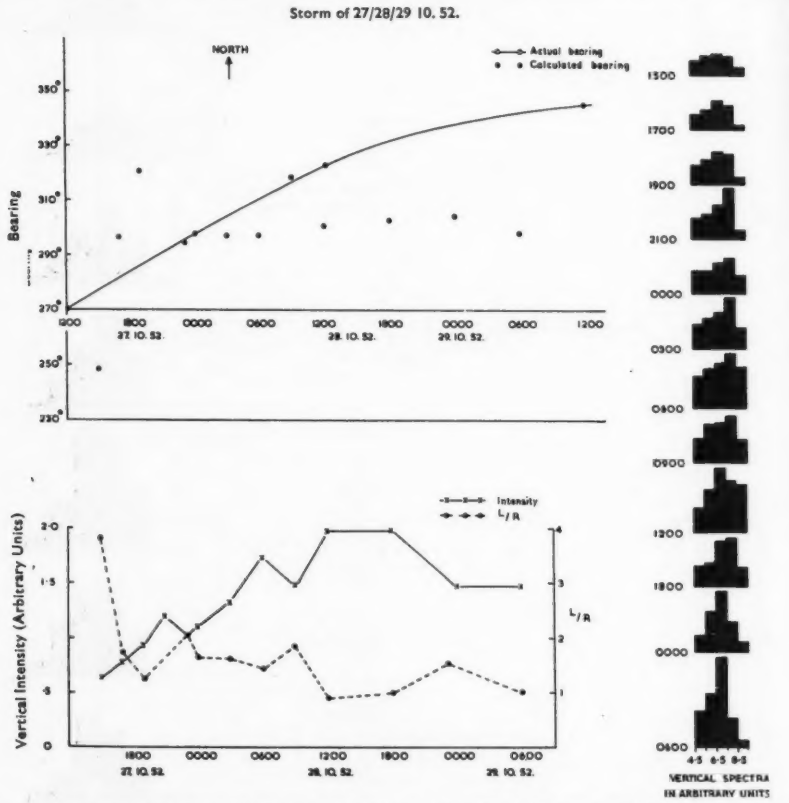

FIG. 5.—*Variation with time of direction of arrival, intensity, L/R ratio and period spectrum for microseisms generated by the storm of* 1952 *October.*

weather situation during its progress. The storm is quite close to the coast and most probably the high waves arriving at the coast act as a second source of microseisms. The spectral peak varies between 6·5 and 8·5s, showing no definite tendency. The L/R ratio, even though showing a slight trend to vary with the intensity and position of the storm, remains fairly constant near about the value of 1·5. The microseisms in this example are clearly due to multiple sources which make storm tracking quite unreliable.

4.3 *Storm of* 1956 *December* (*Figure* 6)

This is studied in two parts, the bearings being evaluated for the whole spectra and the 6–7s band separately. It is quite clear that direction measurements using

total correlation are not able to follow the storm even though there is a tendency to point at a mean direction of 310°. The 6–7s band follows the storm track until 0300 on December 10. Thereafter the apparent generating source moves more towards the South and does not follow the storm movement. From 1200 on the same day there is a sharp rise in intensity accompanied by a spectral shift towards the longer period. This can be due to two causes: (1) intensification of the storm; (2) the arrival of swell on the coast, acting as a second near source of microseisms. The

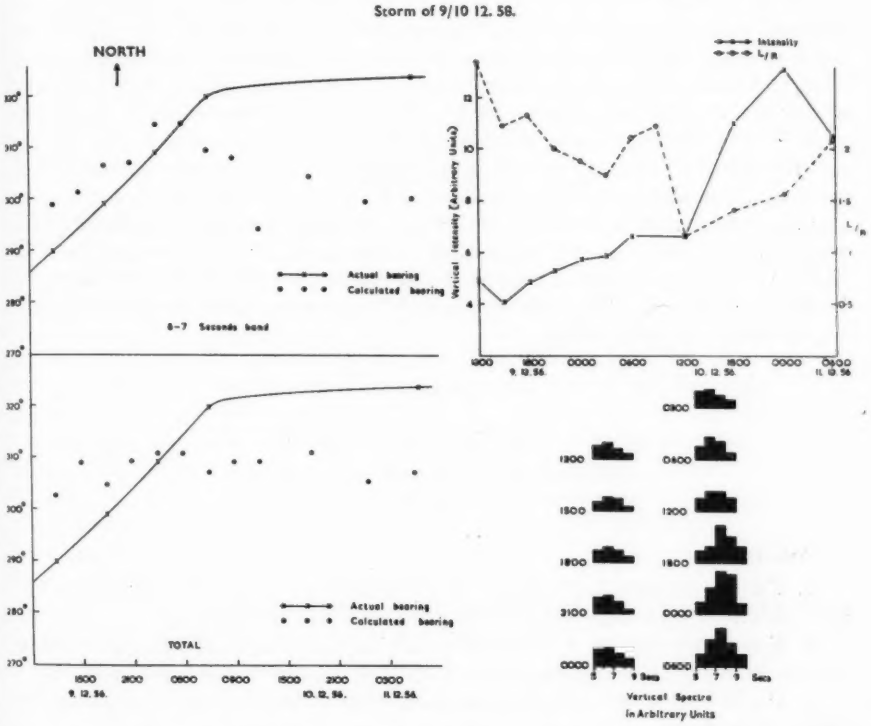

FIG. 6.—*Variation with time of direction of arrival, intensity, L/R ratio and period spectrum for microseisms generated by the storm of* 1956 *December.*

storm has been gradually intensifying but the bearings become inconsistent only at this time. Hence the second alternative looks more likely. But the swell from the storm under consideration could not have reached the coast. It was found however that there was a storm of centre pressure 959 mb off the coast of Greenland about 36 hours before this rise in intensity was noticed, which is the right time for the sea waves to have reached the British coast. The L/R ratio again tends to be affected inversely by the intensity of the storm. It is noticeable that the use of longer records has reduced considerably the statistical fluctuations in the estimated bearings.

It is interesting to note that the calculated bearings for the 6–7s band deviates from the storm position by 9° when the storm is at 290° and gradually reduces to 0° when the storm is at 315°. This may be a case of refraction.

5. A test of the results

Using equations (1), (2) and (3) we get:

$$\frac{\overline{x^2}+\overline{y^2}}{\overline{z^2}} = k^2\left(1 + \frac{\overline{L^2}}{\overline{R^2}}\right) \qquad (13)$$

This equation can be used to evaluate k, the Rayleigh wave constant. Lee (1932, 1934) has evaluated this quantity in detail, both theoretically and practically, for Rayleigh wave propagation through various geological structures. The theoretical value of this quantity for a homogeneous medium without any superficial layers is 0·68, and for Kew Observatory Lee arrived at a theoretical value of 0·95, assuming a layer of limestone over granite. All his values were for microseisms of 2π seconds period. The Rayleigh wave constant varies rapidly with period in the microseism range and hence an accurate estimation of this quantity is not attempted at this stage. k has been evaluated for microseisms from the storms of 1951 and 1952, assuming the vertical component to be twice as sensitive as the horizontals. (This assumption is very nearly true for Kew Observatory.) The values are given in Table 3 in ascending order of magnitude. It can be seen that most of the values lie between 0·6 and 0·8. This fair constancy is a check on the assumptions regarding the nature of microseisms.

Table 3

Rayleigh Wave Constant

0·36, 0·61, 0·64, 0·64, 0·65, 0·67, 0·67, 0·72
0·73, 0·75, 0·76, 0·78, 0·79, 0·91, 0·95, 1·01

6. Conclusions

1. For storms with fairly simple meteorological situations the new method of using Rayleigh wave correlations gives good reliability for tracking.

2. For storms close to the coast the tracking becomes unreliable, presumably because microseisms are generated both at the storm centre and coastal regions.

3. Use of long length records reduces appreciably the statistical fluctuations in estimated bearings.

4. A narrow spectral band at the maximum of the spectrum tends to follow the storm centre more closely than the total spectrum. This indicates that microseismic sources of different periods can exist independently. Further detailed investigation is required before arriving at general conclusions.

5. The Love wave to Rayleigh wave ratio seems to be affected by the storm intensity and the bearing. It shows a tendency to decrease as the storm intensifies. It has already been mentioned that the Love waves in the horizontal components have very little correlation even though they give an appreciable contribution to the total microseismic intensity. Hence the decrease of Love wave to Rayleigh wave ratio as the storm intensifies is presumably because the storm generates mainly Rayleigh waves, the Love waves contributing a kind of noise to the total intensity. The detailed effect of random Love waves on the apparent direction of arrival of microseisms is worth investigating.

6. There are indications of spectral shift attributable either to the intensification of the microseismic source, or the arrival of swell on the coast.

7. Acknowledgments

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National Institute of Oceanography,
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Energy Release in Earthquakes

Leon Knopoff

(Received 1957 September 30)

Summary

The notion of applying methods of static elasticity to the study of energy differences in two states of a given structure was extended by Starr to yield the solution to problems involving the propagation of cracks in shear fields. This technique may be modified to include the solution of the problem of the energy release upon the introduction of a tear fault, such as the San Andreas fault, into an otherwise homogeneous medium subjected to a uniform shear stress. The idealized properties of such a fault are a structure elongated compared with its depth and a strike-slip motion along the fault. This configuration, by symmetry, may be imaged in the Earth's surface so that the problem is reducible to that of a strip fault of infinite length in a homogeneous, isotropic, elastic, infinite medium. The medium is subjected to a uniform shear stress at infinity and the shear stress is assumed to vanish upon the strip. This two-dimensional problem has a vector solution rather than a tensor one, and thus it has an analogue in the electrical problem of a perfectly conducting strip placed in a uniform electric field or that of a strip obstacle placed in a uniform hydrodynamic stream field. The stress distribution and the relative motion throughout the medium before and after faulting can be obtained. For the case of the 1906 San Francisco earthquake, this model yields an energy difference of the shear fields before and after faulting of 4×10^{23} ergs. This value must exceed the elastic wave energy.

1. Introduction

The methods of elastostatics have been applied with some success to the problems of rupture in a solid. The general procedure is to compare the solid in two states: the state of the solid in a uniform, flawless condition and the state of the solid after a crack has been introduced. The procedure is convenient since only problems in static elasticity need be solved for the above comparison; no properties of the motion of the solid during the formation or extension of the flaw are required.

Using this general technique, Griffith (1921) obtained a solution for the energy released upon the formation of a tensile crack, that is to say one formed at right angles to the minimum principal stress. From this elastostatically computed energy, Griffith was able to derive conditions for the propagation of the crack.

Starr (1928) found the energy release and the conditions for the propagation of a shear crack, i.e. one formed parallel to the maximum shear stress in a body. Starr's geometry was that of a crack in the shape of a long thin strip, the elongation

being in the z-direction of a cartesian coordinate system (Figure 1), the strip having its width dimension parallel to x, and the normal to the plane of the strip pointing in the y-direction. The applied stress was the shear stress τ_{xy}. This stress tends to move the faces of the crack parallel to the width or x-dimension. The crack is thus elongated at right angles to the shear direction and the crack extends itself in the direction of the shear.

In this paper we consider the energy release upon the introduction of another type of shear crack, in this case a crack elongated parallel to the shear direction.

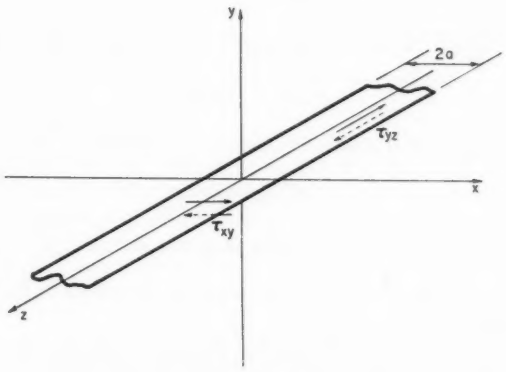

FIG. 1.

The applied stress is the shear stress τ_{yz} for the crack geometry and the coordinate system already considered. In this case all particle motions are parallel to the long dimension of the crack. Since this crack is already of infinite length in the direction of motion, the displacement conditions at the ends of the crack cannot be obtained; hence we have no criteria for the extension of such a crack. We do not attempt to indicate the mechanism of origin of such a crack.

Although originally intended for the problem of the extension of flaws in metals, Starr's solution can be directly applied to faulting in the Earth. The differences are those of the dimensions of the flaws, the energies of the shear fields and the elastic constants of the materials. The model is applicable to normal and thrust faults at depth in the Earth.

The problem considered in this paper is also a model of earth faulting. In this case we solve the problem of the strike-slip fault. As will be seen, the plane $x = 0$ through the centre of the strip and at right angles to it is a plane of zero normal stress. Thus we can solve either the problem of a strike-slip fault at great depth or one which intersects the surface of the Earth.

2. Electric-elastic analogy

Problems in static elasticity, although tensor problems interrelating dilatations and rotations, have vector solutions in certain special cases. In this paper we shall investigate one circumstance in which tensor elastic problems are reducible to vector elastic problems. This reduction is useful because of the wealth of solutions for problems in electrostatics, whose direct application may be advantageously taken to the problems of elasticity if the analogy is formalized.

Consider a shear field in a homogeneous, isotropic, elastic medium such that

the displacement vector $\mathbf{U}$ everywhere points in the z-direction. Let $\mathbf{A}$ be the rotation vector

$$\mathbf{A} = \operatorname{curl} \mathbf{U}. \tag{1}$$

In the absence of dilatations, i.e. in a pure shear field, the differential equation of static elasticity

$$(\lambda+2\mu) \operatorname{grad} \operatorname{div} \mathbf{U} - \mu \operatorname{curl} \operatorname{curl} \mathbf{U} = 0, \tag{2}$$

reduces to

$$\operatorname{curl} \mathbf{A} = 0. \tag{3}$$

λ and μ are the Lamé moduli of elasticity. Thus $\mathbf{A}$ is derivable from a scalar potential

$$\mathbf{A} = -\operatorname{grad} \phi, \tag{4}$$

in which the scalar potential ϕ satisfies Laplace's equation

$$\nabla^2 \phi = 0, \tag{5}$$

since

$$\operatorname{div} \mathbf{A} = 0 \tag{6}$$

from equation (1). Hence the rotation vector $\mathbf{A}$ describes both a solenoidal and an irrotational field.

Superficially the vector field $\mathbf{A}$ satisfies the same differential equations as does the electrostatic field in a charge-free situation. If the vector $\mathbf{A}$ can be shown to satisfy the same boundary conditions as the electric field vector $\mathbf{E}$ then the correspondence will be complete and the solutions for certain electrostatic boundary value problems can be applied to the corresponding elastostatic boundary value problems.

We propose to introduce a surface S, in the elastostatic case, which will act as a singular region in the shear field. We shall require that the shear stress vanishes everywhere on this surface,

$$\tau_{yz} = 0 \text{ on } S.$$

Hence, we require the shear strain $e_{yz} = 0$ on S. Now

$$e_{yz} = \tfrac{1}{2}(\partial U_y/\partial z + \partial U_z/\partial y). \tag{7}$$

But by hypothesis we have a displacement field pointing only in the z-direction. Thus

$$\partial U_z/\partial y = 0 \text{ on } S. \tag{8}$$

Now

$$\mathbf{A} = (\partial U_z/\partial y, -\partial U_z/\partial x, 0). \tag{9}$$

If now S is assumed to occupy parts of the planes $y =$ constant, then $\mathbf{A}$ is normal to S on S. This corresponds to the electrostatic boundary condition upon the electric field in the vicinity of a perfect conductor. The analogy is now complete. In order that $\mathbf{A}$ shall never have a z-component, we only solve two-dimensional problems so that z is not a variable in the problem.

It is now an elementary task to determine the remaining properties of the analogy. These are given in Table 1. $\mathbf{n}$ is a unit vector normal to a surface infinite in the z-direction; $\mathbf{z}_1$ is a unit vector in the z-direction.

Table 1

Two-dimensional analogies

Electric Quantity		*Elastic Quantity*	
$\mathbf{E}$	Electric field	$\mathbf{A}$	Rotation vector
ϵ	Dielectric constant	μ	Shear modulus
ϕ	Potential of field	ϕ	Potential of rotation
ψ	Stream function	U_z	Displacement vector (unidirectional)
$\mathbf{D}\times\mathbf{z}_1$	Rotation of Displacement	$\boldsymbol{\tau}$	Stress vector $\boldsymbol{\tau} = \tau_{xz}\mathbf{x}_1 + \tau_{yz}\mathbf{y}_1$
$\mathbf{E}\times\mathbf{z}_1$	Rotation of Electric Field	$\mathbf{e}$	Strain vector $\mathbf{e} = \mathbf{e}_{xz}\mathbf{x}_1 + \mathbf{e}_{yz}\mathbf{y}_1$
$\mathbf{D}$	Displacement	$\mathbf{z}_1\times\boldsymbol{\tau}$	Rotation of stress
σ	Surface charge	$\mathbf{z}_1\times\boldsymbol{\tau}\,.\,\mathbf{n}$	Component of stress
$\mathbf{E}\times\mathbf{n} = 0$	Perfect conductor	$\boldsymbol{\tau}\,.\,\mathbf{n} = 0$	Perfectly weak crack

3. The field around a strip

As an example of the calculation, consider a uniform shear field at infinity and a cut in the finite domain in the shape of a strip of infinite length, of width $2a$ and occupying part of the plane $y = 0$ as shown on Figure 1. Let the strip extend to infinity in the z-direction. At infinity, or in the absence of the strip, let the displacement be

$$\mathbf{U} = (0, 0, A_0 y). \tag{10}$$

The rotation vector in the absence of the strip is

$$\mathbf{A} = (A_0, 0, 0). \tag{11}$$

Thus we have set up the problem of a uniform field perturbed by a strip of infinite length whose short dimension is parallel to the unperturbed field.

This problem, as is well known, is solved in electrostatics or in hydrodynamics by a conformal transformation. The potential is (Smythe 1950, p. 92)

$$\phi = -A_0 \operatorname{Re}(w^2 - a^2)^{\frac{1}{2}}, \tag{12}$$

where $w = x + iy$. This is seen to satisfy the condition at infinity, since for $|w| \gg a$, $\phi = -A_0 x$ leading to $\mathbf{A} = (A_0, 0, 0)$ at infinity. On the strip, $w = x$ where $|x| < a$. Hence, $\phi = 0$ on the strip. Thus the strip is an equipotential surface and the $\mathbf{A}$ lines are everywhere normal to it.

We are now in a position to solve for several features of the field in the presence of the strip.

3.1 *The displacement field.*

From Equation (4)

$$\mathbf{A} = (-\partial\phi/\partial x, -\partial\phi/\partial y, 0). \tag{13}$$

Comparing equation (13) with equation (9), we see that the Cauchy-Riemann conditions are satisfied. Hence U_z is orthogonal to ϕ in the complex w-plane. Thus, if ϕ is given by equation (12), the displacement anywhere is

$$U_z = A_0 \operatorname{Im}(w^2 - a^2)^{\frac{1}{2}}. \tag{14}$$

In the plane $y = 0$,

$$U_z = 0, \qquad |x| > a$$
$$U_z = A_0 (a^2 - x^2)^{\frac{1}{2}} \qquad |x| < a.$$

At the origin, $U_z = A_0 a$. Of course, on the opposite side of the strip, $U_z = -A_0 a$.

In the plane $x = 0$,

$$U_z = \pm A_0(y^2+a^2)^{\frac{1}{2}} \qquad y \gtrless 0.$$

This, logically, approaches the value $\pm A_0 y$ as $|y| \to \infty$.

We may inquire into the distance from the strip in the plane $x = 0$, that the relative displacement falls to half the value at the crack. The displacement in the absence of the strip is $A_0 y$. In the presence of the strip it is $A_0(y^2+a^2)^{\frac{1}{2}}$. The relative displacement is

$$A_0[(y^2+a^2)^{\frac{1}{2}}-y]. \tag{15}$$

At the strip this has the value $A_0 a$. Solving the equation

$$(y_0^2+a^2)^{\frac{1}{2}}-y_0 = a/2, \tag{16}$$

we obtain a distance $y_0 = 3a/4$ for the "half-displacement" point.

3.2 *The strain field*

The only strain components are

$$e_{zy} = \tfrac{1}{2}\partial U_z/\partial y = -\tfrac{1}{2}A_0 \operatorname{Re} w(w^2-a^2)^{-\frac{1}{2}} \tag{17}$$

$$e_{zx} = \tfrac{1}{2}\partial U_z/\partial x = \tfrac{1}{2}A_0 \operatorname{Im} w(w^2-a^2)^{-\frac{1}{2}}, \tag{18}$$

all other terms in the strain tensor are zero either because of the nature of the displacement vector or because of the two-dimensionality of the problem.

On the plane $y = 0$,

$$\begin{aligned} e_{zy} &= 0 && |x| < a \\ &= -\tfrac{1}{2}A_0 x(x^2-a^2)^{\frac{1}{2}} && |x| > a. \end{aligned}$$

The first of these two expressions is consistent with the boundary condition (8).

On the plane $x = 0$, $e_{zx} = 0$. Thus the surface $x = 0$ is a free surface. If we wish to consider the present problem as a model of displacement due to faulting, the surface $x = 0$ may be taken as the free surface of the Earth, and the crack $y = 0$, $0 < x < a$, may be taken as the fault. The total relative displacement between the two sides of the fault is $2A_0 a$ at the surface of the crack.

3.3 *Energy loss by insertion of the crack*

We can now inquire into the energy loss by reduction of the stress in the vicinity of the crack. The energy in a medium having only shear strain is

$$W = \tfrac{1}{2}\mu \int_V (\operatorname{curl} \mathbf{U})^2 dv$$

integrated over all space where dv is an element of volume and μ is the shear modulus. Thus the energy difference between the two static states of strain is

$$W = \tfrac{1}{2}\mu \int_V [A_0^2 - |\mathbf{A}|^2] dv, \tag{19}$$

where $\mathbf{A}$ is the rotation in the case of the medium in the presence of the crack. It is at this stage that the value of the electrical analogy is fully appreciated. If the

value of **A** is inserted in equation (19) from the expressions for the potential given in equations (4) and (12), the integral to be evaluated diverges. That is to say, it leads to an infinite result when integrated over one coordinate and to a zero result when integrated over the other coordinate. To avoid this singular circumstance two procedures are available and both draw heavily upon experience obtained in the corresponding electrical problem. Stratton (1941, p. 118) shows that the integral of equation (19) may be modified and written in the form

$$\Delta W = \tfrac{1}{2}\mu \int_{V_0} A_0^2 dv + \tfrac{1}{2}\mu \int_{V_1} (A_0 - A_1)^2 dv, \tag{20}$$

where V_0 is the volume of a conductor inserted into the field and where V_1 is the volume remaining after this insertion has taken place. A_0 is the field distribution before the insertion of the conductor; A_1 is the field distribution after insertion. In the present case $A_0 = (A_0, 0, 0)$ and $V_0 = 0$ since the strip encloses no volume. V_1 now occupies all space. Thus if dS is an element of area in the x–y plane, the energy difference per unit length* is

$$\Delta E = \tfrac{1}{2}\mu \int_S [(\Delta A_x)^2 + (\Delta A_y)^2] dS, \tag{21}$$

where

$$\Delta A_x = A_0(1 - \operatorname{Re} w(w^2 - a^2)^{-\frac{1}{2}}, \tag{22}$$

and

$$\Delta A_y = A_0 \operatorname{Im} w(w^2 - a^2)^{-\frac{1}{2}}. \tag{23}$$

It is now convenient to solve this problem in elliptical coordinates (Stratton, pp. 53–54). We let

$$x = a\xi\eta \tag{24}$$

$$y = a(\xi^2 - 1)^{\frac{1}{2}}(1 - \eta^2)^{\frac{1}{2}}. \tag{25}$$

The element of area is

$$dS = \frac{a^2(\xi^2 - \eta^2) d\xi d\eta}{(\xi^2 - 1)^{\frac{1}{2}}(1 - \eta^2)^{\frac{1}{2}}}.$$

After much algebra

$$\Delta A_x = A_0 \left\{ \frac{\xi^2 - \eta^2 - \xi(\xi^2 - 1)^{\frac{1}{2}}}{\xi^2 - 1} \right\} \tag{26}$$

$$\Delta A_y = -A_0 \frac{\eta(1 - \eta^2)^{\frac{1}{2}}}{\xi^2 - \eta^2}. \tag{27}$$

Substituting these quantities into equation (21),

$$\Delta E = 2\mu A_0^2 a^2 \int_0^1 \frac{d\eta}{(1 - \eta^2)^{\frac{1}{2}}} \int_1^\infty \frac{2\xi^2 - 1 - 2\xi^2(\xi^2 - 1)^{\frac{1}{2}}}{(\xi^2 - 1)^{\frac{1}{2}}} d\xi. \tag{28}$$

* We use the notation W to represent total energy and E to represent energy per unit length.

This integral is convergent. The result of the integration is

$$\Delta E = \tfrac{1}{2}\pi\mu A_0^2 a^2 \tag{29}$$

per unit length.

The second technique of obtaining this result is to use the form of equation (19) where it is converted to an integral only over the volume of the inserted conductor. This expression is (Stratton, p. 113)

$$\Delta W = \tfrac{1}{2}\int_{V_0} (\mu'-\mu)\mathbf{A}'\cdot\mathbf{A}_0 dv \tag{30}$$

where instead of inserting a conducting strip, we insert an elliptical cylinder into the region where the new cylinder has a shear modulus μ' differing from that of the surrounding medium μ. The field inside the cylinder, $\mathbf{A}'$ is given by Smythe (p. 97) as

$$\begin{cases} A_x' = A_0(a+b)(a+b\mu'/\mu)^{-1} \\ A_y' = A_2' = 0. \end{cases} \tag{31}$$

The energy difference is

$$\Delta E = \frac{1}{2}\cdot\frac{\mu'-\mu}{a+b\mu'/\mu}(a+b)A_0^2 abn, \tag{32}$$

since the field on the inside of the elliptic cylinder is uniform and parallel to the external field $\mathbf{A}_0$. a and b are the semi-major and semi-minor axes of the cylinder. Now allowing the cylinder to become perfectly conducting by allowing the shear modulus μ' to become infinite,

$$\Delta E = \tfrac{1}{2}\pi(a+b)aA_0^2\mu$$

and finally allowing the cylinder to become infinitely thin, the energy necessary to insert the strip of width $2a$ is

$$\Delta E = \tfrac{1}{2}\pi a^2 A_0^2\mu \tag{33}$$

per unit length in agreement with the result of equation (29).

4. Application to the San Francisco earthquake, 1906

Let the San Andreas fault be represented by a plane vertical crack of length L, of infinitesimal width, and of depth to the bottom of the crack a. Before faulting let the region be in a uniform shear field $\mathbf{U} = (0, 0, A_0 y)$ and after faulting let the crack now represent a surface of zero shear stress. The surface of the Earth is the plane $x = 0$. The surface trace of the fault shows an offset $s = 2A_0 a$. The energy release is $\tfrac{1}{4}\pi\mu A_0^2 a^2 L = (1/16)\pi\mu s^2 L$. We neglect end effects due to the termination of the fault. For the values (Reid 1910, p. 22) $\mu = 3\times 10^{11}$dyn/cm^2, $s = 4$m, $L = 435$ km, we obtain the energy $\Delta W = 4\times 10^{23}$ ergs. This figure must include both the seismic wave energy and the energy connected with non-elastic effects such as heat and plastic deformation.

The displacement on the crack is a maximum at the surface of the Earth and falls to zero at the lower edge. Thus there is no discontinuity in the shear strain at the lower edge.

The depth of faulting (i.e. the half-width a of the crack) can be obtained by fitting the triangulation observations (Hayford & Baldwin 1908, pp. 133–134) to the curve given in equation 15. Assuming, as Reid does, that the motion of the Farallon Islands represented a regional offset of the portion of the Earth's crust lying to the west of the fault, the 33 points to which the curve is to be fitted are tabulated in Table 2. We obtain, by a least squares technique, the curve

$$U_y = \pm 0{\cdot}72[(y^2 + 3{\cdot}2^2)^{\frac{1}{2}} - |y|]$$

Table 2

Triangulation data for San Francisco Earthquake, 1906

Number of Points	Distance of Stations from fault	Average Displacement	Displacement corrected for regional offset
10	1·5 km.	1·54 m.	1·54 m.
12	2·0	2·95	1·17
3	4·2	0·86	0·86
7	5·8	2·38	0·60
1	6·4	0·58	0·58
1	37·	1·78	0 (Farallon I. station)

where U_y is the offset in metres and y is the distance from the fault in kilometres. The plus and minus signs refer to the relative displacements on the two sides of the fault. To these values, the regional offset must be added for the stations west of the fault. The relative displacement at the fault is thus seen to be 4·5m compared to Reid's value of 4m referred to above. The depth to the bottom of the crack is now seen to be 3·2 km, a value considerably less than depths usually assumed for California earthquakes and for the San Francisco earthquake in particular.

It should be noted that the model used here assumes a uniform shear modulus everywhere, including the material at great depth in the Earth, and including the region very close to the fault.

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Geomagnetic Pulsations and the Earth's Outer Atmosphere

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Summary

Hydromagnetic oscillations of the Earth's ionized outer atmosphere along the geomagnetic lines of force are considered. The observational evidence of world-wide geomagnetic pulsations yields the distribution of ionic density in the outer atmosphere extending beyond the ionosphere. It is found that the ion density is about 10^3 per cm^3 at a distance of a few earth radii decreasing exponentially to a value of the order of 5 per cm^3 in interplanetary space. A theoretical consideration of the temperature of the outer ionosphere is also discussed.

1. Introduction

Little was known about the nature of the atmosphere beyond the Earth's ionosphere until quite recently. Exploration by radio waves and the study of the emission spectra of night air glow and aurorae give considerable information about the ionosphere, but yield little on the outer ionosphere at greater heights. However, some evidence indicating the existence of an extending outer atmosphere has been known for a long time.

Twilight flashes of the red oxygen lines can be traced up to a height of 1300 km (Elvey 1948). Auroral streamers, particularly those in the sunlit atmosphere, have also been observed up to heights as great as 1000 km (Störmer 1937). Astrophysicists, on the other hand, have been concerned with interstellar gases. Zodiacal light indicated evidence of the scattering of solar rays by extensive clouds of particles in interplanetary space, and Siedentopf, Behr & Elsasser (1953) showed that the density of interplanetary gas is as much as 600 hydrogen atoms per cm^3. Recent developments in radio astronomy, especially the observation of the 21 cm line of hydrogen, provides much information about hydrogen clouds. It is well established that solar corpuscular streams ejected from the Sun are responsible for aurorae, magnetic storms and ionospheric disturbances. Furthermore, certain cosmic ray intensity variations are now generally regarded as being due to the movements of interplanetary clouds (Simpson 1956, Forbush 1956).

Considerable developments have taken place in the last few years. Storey (1953) has shown that "whistling atmospherics" consist of waves travelling along the Earth's magnetic lines of force through the ionized outer atmosphere, and

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further investigation should reveal important information on the nature of the Earth's outer atmosphere. Thus theattention of many scientific disciplines has been concentrated in this field. Further insight into the problem has been given by Dungey (1954) who proposed the theory that geomagnetic pulsations might be caused by hydromagnetic oscillations of the Earth's ionized outer atmosphere.

Geomagnetic pulsations are small regular fluctuations of the geomagnetic field, the oscillations lasting for an hour or more. The period varies from a few seconds to about three minutes and the amplitude is usually several gammas. They occur not only during storms but also on otherwise quiet days. Although the history of the study of geomagnetic pulsations is very old, observational knowledge is still inconsistent and sparse. Kato & Watanabe (1957) gave an extensive survey of the observational knowledge including recent interesting results of their own comprehensive researches. In their paper, they emphasize the necessity of a systematic world-wide morphology of pulsations, stressing the value of the International Geophysical year.

Since this problem seems to offer great promise for a clearer insight into the physical nature of the outer atmosphere, some preliminary investigations have been attempted. In the present paper, a theoretical consideration is given of the hydromagnetic oscillations of the ionized outer atmosphere as a cause of geomagnetic pulsations. The latitude variation of the period of world-wide pulsations is also obtained. Applying this observational result to the theoretical model, the distribution of ionic density in the atmosphere is determined. Some discussion of the physical nature of the outer atmosphere is also given.

2. Hydromagnetic oscillations of the outer atmosphere

An ionized gas is capable of a wide variety of oscillatory motions. In general these oscillations may be classified into three particular types of waves, electromagnetic waves, hydromagnetic waves and electrostatic waves (plasma oscillations). For the investigation of geomagnetic pulsations, hydromagnetic waves are the most important since the frequency range corresponding to geomagnetic pulsations is far below the gyro-frequency of positive ions. In a hydromagnetic wave, the positive ions provide the inertia of the oscillation, while the restoring forces are largely magnetic. Thus the oscillation may be regarded as waves in the magnetic line of force, which behaves as a stretched string allowing the propagation of transverse waves.

The possibility of the existence of such waves was first suggested by Alfvén (1942). Waves, known as Alfvén waves, are propagated along the magnetic lines of force with a velocity whose square is equal to the magnetic stress density divided by the ionic density ρ_i,

$$V = \frac{H}{(4\pi\rho_i)^{\frac{1}{2}}}. \tag{1}$$

As the magnetic energy density of the gas becomes large compared with the kinetic energy density, all hydromagnetic motions reduce to hydromagnetic waves. Thus in the outer terrestrial atmosphere, a small disturbance, which may possibly be due to the invasion of ionized clouds into the Earth's magnetic field, will excite hydromagnetic perturbations and propagate as Alfvén waves along the geomagnetic lines of force. This possibility has been considered by Akasofu (1956) and the general solution of outer atmospheric oscillations has been obtained under particular conditions.

In the present study, the oscillation of hydromagnetic waves induced along the geomagnetic lines of force is treated by an analogy to stretched strings; the period along a certain geomagnetic line of force corresponding to co-latitude θ_0 is given by

$$T_p = \int_{\theta_0} \frac{2ds}{V}. \tag{2}$$

As illustrated in Figure 1, the integration is along the line of force, which cuts the Earth's surface at θ_0. The geomagnetic field is assumed to be a centred dipole. The equation of the line of force is then given by

$$\nu = \frac{r}{a} = \frac{\sin^2\theta}{\sin^2\theta_0} \tag{3}$$

and

$$H = \frac{C_0}{\nu^3}(1+3\cos^2\theta)^{\frac{1}{2}} \tag{4}$$

where ν is the distance from the centre of the Earth measured in units of earth radii, and C_0 is 0·3 Γ. Then equation (2) becomes

$$T_p = \frac{8\pi^{\frac{1}{2}}}{C_0 \sin^8\theta_0} \int_{\theta_0}^{\frac{1}{2}\pi} \rho_i^{\frac{1}{2}} \sin^7\theta \, d\theta. \tag{5}$$

Since Tp can be observed as the period of geomagnetic pulsations in different latitudes (θ_0), the problem is to solve this integral equation in order to determine the unknown function ρ_i. However, a formal mathematical solution is complicated. Therefore, assuming a particular mathematical form for the function ρ_i, Tp is calculated by numerical integration.

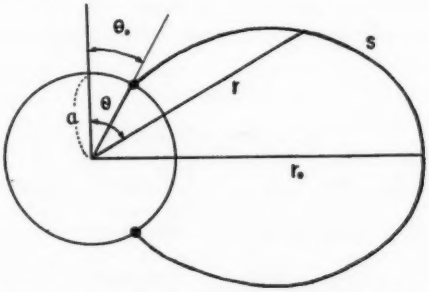

FIG. 1.—*Geometry of the geomagnetic lines of force.*

Following Dungey (1955) the outer atmosphere is assumed to consist chiefly of ionized hydrogen in thermal and hydrostatic equilibrium under gravity. Then

$$\rho_i = m_H N \tag{6}$$

and

$$N = N_s \exp\left(\frac{a}{H_0 \nu}\right), \tag{7}$$

where N_s is the ionic density in interplanetary space far from the Earth, and H_0

is the scale height of the ionization at the given temperature and at the value of gravity which is found at the Earth's surface. Putting $\alpha = a/H_0$,

$$T_p = \frac{8a(\pi m_H)^{\frac{1}{2}}}{C_0 \sin^8 \theta_0} N_s^{\frac{1}{2}} \int_{\theta_0}^{\frac{1}{2}\pi} \exp\left(\frac{\alpha \sin^2 \theta_0}{2 \sin^2 \theta}\right) \sin^7 \theta \, d\theta. \tag{8}$$

This equation can also be written in the form

$$\ln T_p = \ln N_s^{\frac{1}{2}} + \ln F(\theta_0, \alpha) \tag{9}$$

and

$$\frac{d}{d\theta_0} \ln T_p = \frac{d}{d\theta_0} \ln F(\theta_0, \alpha). \tag{10}$$

From the observed values of T_p, which are functions of θ_0, α and N_s can be determined by solving the simultaneous equations (9) and (10).

A graph of the calculated curves of $T_p/N_s^{\frac{1}{2}}$ for different values of α is shown in Figure 2. The curves are plotted against $(\cos \Phi)^{-2}$ since, with this abscissa, the relationship is nearly linear for small values of α. $(\cos \Phi)^{-2}$ is proportional to the maximum distance from the Earth's surface of the geomagnetic line of force through geomagnetic latitude Φ.

3. Observational results of geomagnetic pulsations

For the systematic study of world-wide geomagnetic pulsations it is necessary to examine the simultaneous records from many magnetic stations and over a sufficiently long period of time. Though it was not possible to fulfil such requirements in the present study, a considerable number of good photostat copies of magnetograms from several stations were fortunately available. These had originally been collected for the purpose of studying the sudden commencements of magnetic storms. Thus about 80 days of records from 1949 to 1953 were used to investigate simultaneous geomagnetic pulsations.

Magnetic stations whose data are used in the present analysis are listed in Table 1.

Table 1

List of magnetic stations

Station	Abbreviation	Φ	Λ	Station	Abbreviation	Φ	Λ
Tromso	Tr	67·1	116·7	Toyohara*	Ty	36·9	203·5
College	Co	64·5	255·4	Onagawa*	On	28·2	206·0
Sitka	Si	60·0	275·4	Toolangi	Tl	−46·7	220·8
Niemegk	Ni	52·2	96·5	Amberley	Am	−47·7	252·5
Cheltenham	Ch	50·1	350·5	Macquarie Is.	Mq	−61·1	243·1
Tucson	Tu	40·4	312·2				

* For these stations, statistical results from other authors and not actual magnetograms were used.

The magnetograms used here are the ordinary records of the H, D and Z components which run 20 mm per hour. Therefore the minimum resolution of the

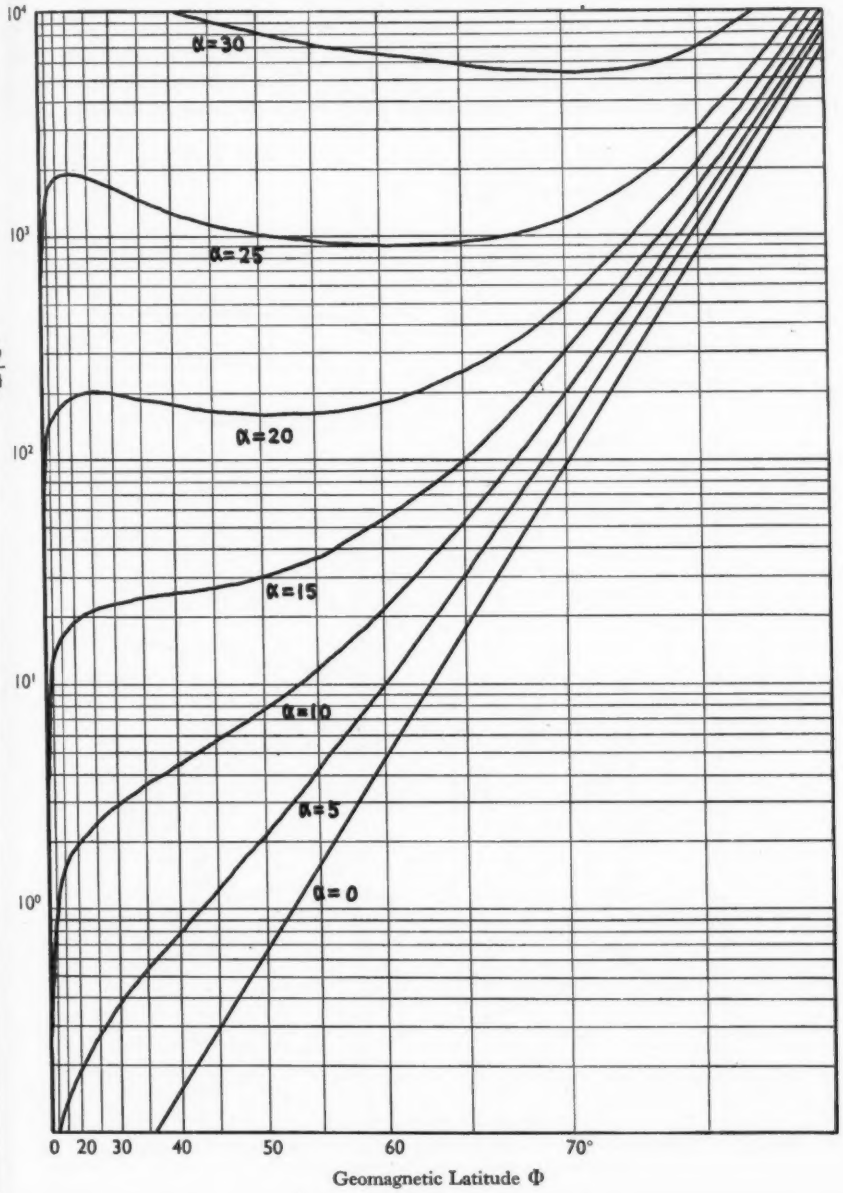

FIG. 2.—*Computed curves of* $T_T/N_0^{\frac{1}{2}}$.

period is of the order of one minute. To find geomagnetic pulsations, special attention was paid to the data from College, Sitka, Cheltenham and Amberley. In particular from the latter two stations, very beautiful pulsations of about one minute period appearing as serrations on the magnetogram were often observed, though for stations in the auroral zone it was rather difficult to identify them owing to additional disturbances. In the present analysis, only those pulsations which

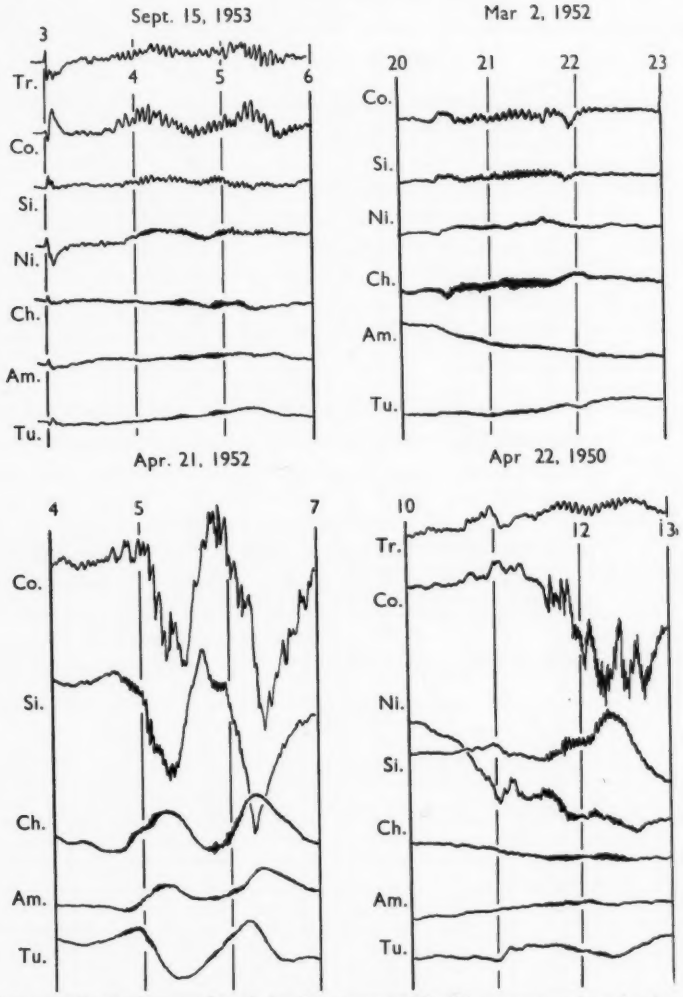

FIG. 3.—*Typical examples of simultaneous world-wide geomagnetic pulsations.*

were observed simultaneously at more than two of the above mentioned stations were accepted.

Typical examples of pulsations are reproduced in Figure 3. Some of them occurred during (magnetically) rather quiet conditions, while others were accompanied by disturbances particularly at the beginning of "bay disturbances". An examination of the data showed that in general the period becomes longer and the

amplitude larger with increasing geomagnetic latitude, although during severe disturbances very rapid oscillations were observed in the auroral zone on several occasions.

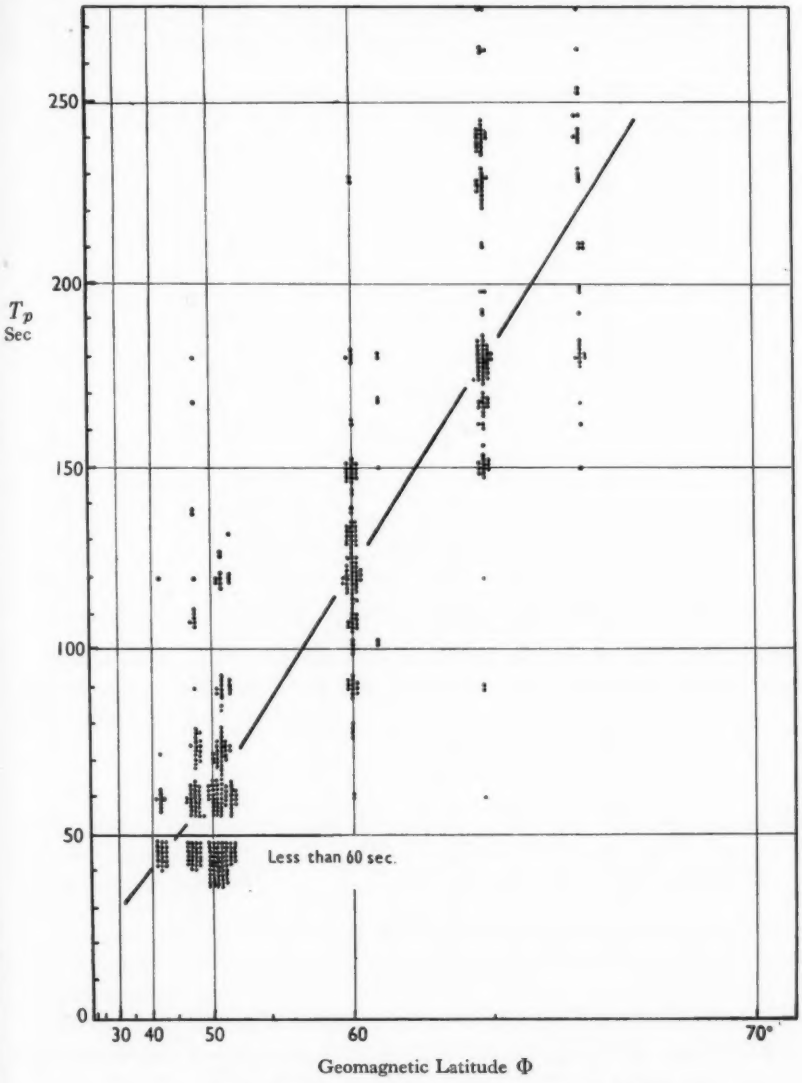

FIG. 4—*Plot of observed periods of geomagnetic pulsations.*

A statistical scatter diagram of the period of pulsation with respect to geomagnetic latitude is shown in Figure 4. Although there is considerable scatter for individual pulsations, the period becomes longer with increasing latitude and, as has been mentioned, an almost linear relationship exists between the period and $(\cos \Phi)^{-2}$.

4. The Earth's outer atmosphere

Applying the observational relationship obtained above, the distribution of the ionic density in the outer atmosphere has been determined. As shown in Figure 5, the best agreement with observational results is obtained when N_s is 3·75 and α is 15. In Figure 5, observational results obtained in Japan by Kato, Ossaka,

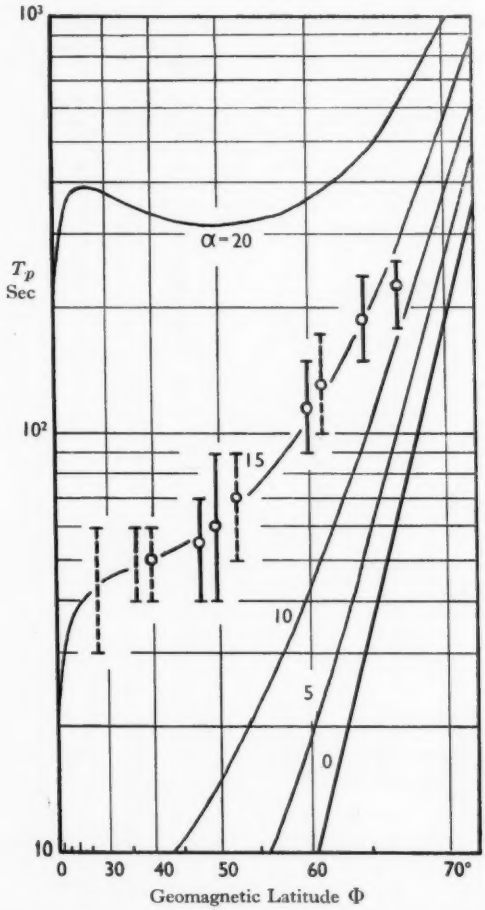

FIG. 5.—*Comparison between observed and theoretical values* T_p *for* $N_s = 3{\cdot}75/cm^3$.

Watanabe, Okuda & Tamao (1955), Kato & Watanabe (1957) and Hatakeyama (1938) are also added.

These values of N_s and α then yield the distribution of the ionic density, which is illustrated in Figure 6. The observational data are obtained mainly from stations whose geomagnetic latitude lies in the range 40° to 65°, the estimated ion distribution from this range corresponding to values of ν between 2 and 5. This portion is indicated by a solid line in the figure.

The ion density is about 10^7 per cm³ at the Earth's surface (i.e. at the top of the ionosphere), and 10^3 per cm³ at a distance of a few earth radii. These values are

in striking agreement with the completely independent results of ionospheric observations, which give a maximum electron density in the F2 layer of about 10^6 per cm³. "Whistling atmospherics" indicate 1000–2000 per cm³ at $\nu = 2 \sim 3$ (Storey 1957). Furthermore, the density in interplanetary space obtained here is of the same order as the value in interstellar space, viz. $N = 1$ atom per cm³ (in spiral arms) as estimated by Spitzer (1954).

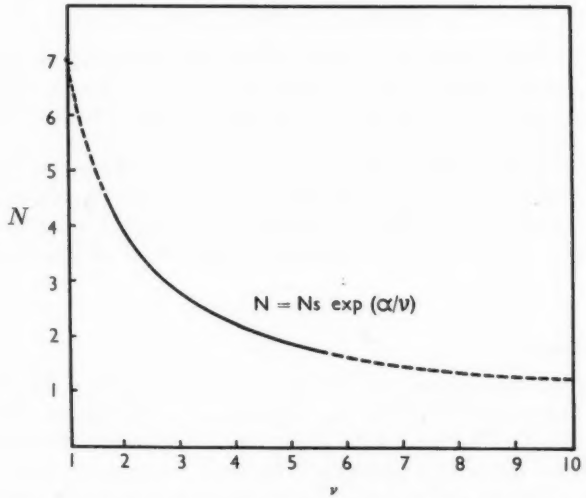

FIG. 6.—*Distribution of ionic density in the outer atmosphere for* $\alpha = 15$, $N_s = 3{\cdot}75/cm^3$.

The temperature of the atmosphere can also be estimated from the value of α. Assuming isothermal hydrostatic equilibrium conditions under gravity, equation (7) can be written in the form

$$\alpha = \frac{a}{H_0} = a \cdot \frac{\mu m_H g_0}{kT} = \frac{a\mu}{0{\cdot}85T}, \qquad (11)$$

where μ is the mean molecular weight and g_0 the value of gravity at the Earth's surface. Taking the value of α equal to 15 gives the scale height $H_0 = 425$ km. Therefore T/μ is equal to 500°K. If the hydrogen gas is fully ionized i.e. $\mu = \frac{1}{2}$, then it gives a temperature of about 250°K.

However, this value of the temperature is certainly too low for ionized hydrogen gas. There is considerable evidence that the temperature of the uppermost regions of the ionosphere is about 1000°–2000°K (Spitzer 1949). The temperature estimated from the scale height does not agree with this. However, there is considerable doubt that the outer atmosphere is in hydrostatic equilibrium governed only by gravity and temperature. The mean free path of the particles becomes extremely large in the outer atmosphere, and also the motion and distribution of the particles, if ionized, will be profoundly influenced by the Earth's magnetic field. Therefore the temperature estimated from the condition of hydrostatic equilibrium is certainly not reliable and it is more likely that the temperature in interplanetary space is well above a few thousand degrees.

In the present study, the cause of the geomagnetic pulsations is attributed to certain ionized clouds which invade the Earth's magnetic field. Since there is some

evidence that geomagnetic pulsations show appreciable 27-day recurrence tendency (Kato & Watanabe 1957), it is almost certain that these clouds are of solar origin. If the density of ionized clouds is assumed to be of the order of 10 per cm^3 and the velocity of the order of 500 km/s, the kinetic energy of the clouds is balanced by the stress energy of the geomagnetic field at the distance of v_0, namely

$$\tfrac{1}{2}\rho V^2 = \frac{H^2}{8\pi}, \tag{12}$$

giving $v_0 = 7$. Hence clouds are stopped where the corresponding geomagnetic line of force cuts the Earth's surface where $\Phi \simeq 67{\cdot}5°$. This explains why the amplitude of geomagnetic pulsations near the auroral zone is large, while decreasing gradually towards low latitudes.

Although the observational material used here is not sufficient to reach a firm conclusion, characteristics of geomagnetic pulsations can be explained fairly consistently. The distribution of ionic density in the outer atmosphere obtained from this analysis also shows good agreement with other independent investigations.

Acknowledgments

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1957 *September* 13.

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Polar Movement Relative to Australia

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(Received 1957 October 28)

Summary

During Upper Proterozoic times the mean geomagnetic pole seems to have been situated near Australia. By the Cambrian it appears to have moved to what is nowadays the region of South Africa passing southwards into the Southern Ocean by Silurian and early Devonian times. In the Upper Carboniferous and Permian the pole lay in the Tasman Sea and Australia was once again in a high geomagnetic latitude. Later the pole moved slowly southwards, reaching the present geographic pole towards the end of the Tertiary.

1. Introduction

The mean position of the geomagnetic poles during any geological epoch may be calculated from estimates of the mean direction of magnetization of rock formations laid down at that time (Creer, Irving & Runcorn 1957). The method of obtaining these estimates has been discussed previously (Fisher 1953, Watson & Irving 1957, Runcorn 1957). These pole positions are given relative to the area in which the formation is situated. If rock formations from successive geological periods are studied in the same general region the path of polar wandering relative to this region is obtained. From palaeomagnetic surveys of some rock formations in Australia the movements of the geomagnetic pole relative to Australia during and since the Upper Proterozoic has been obtained in broad outline. The data are very far from complete but sufficient is known for certain general points to be made. The path of movement is known in some detail from Carboniferous to Recent times and with less certainty for periods prior to this. The stability of the magnetization of several of the rock formations studied has been demonstrated (Irving 1956, 1957a) by tests of the type described by Graham (1949).

Samples have been collected from rock formations distributed over a wide area of the Commonwealth (Figure 1). Most of the samples have been collected by the authors with invaluable geological guidance from Dr W. R. Browne of Edgecliff, Sydney, Professor S. W. Carey and Mr M. R. Banks of the University of Tasmania, Dr O. P. Singleton of the University of Melbourne, and Dr A. A. Opik and Mr L. C. Noakes of the Bureau of Mineral Resources. In addition, many samples have been sent to us by officers of the Bureau of Mineral Resources by arrangement with Dr N. H. Fisher, and the Geological Survey of Western Australia by arrangement with Mr H. A. Ellis. The following officers of these organisations have collected specimens for us: Messrs M. A. Randall, R. Stuart and N. Mackay of the Bureau of Mineral Resources, and Mr L. de la Hunty of the Geological Survey of Western Australia. The specimens of Brisbane tuff

were sent to us by Mr I. Macleod of the University of Queensland. We wish to express our gratitude to these people for their very great help, and also to Professor J. C. Jaeger, Head of this Department, for many helpful suggestions and criticisms.

A list of the formations studied in Australia with their mean directions of magnetisation and attendant pole positions is given in the table. Each result is numbered in order of age from the oldest to the youngest, the numbering in the

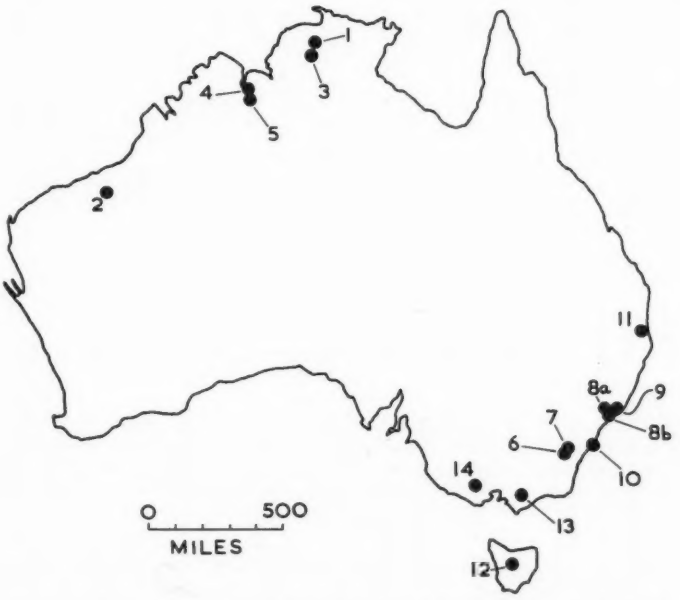

FIG. 1.—*Locations of the rock formations sampled for palaeomagnetic work.* The formations are numbered as in the table.

table and the figures being consistent. Correction is always made for geological dip so that the directions are given relative to the bedding (or, in the case of igneous rocks, the bedding of adjacent sediments) which is assumed horizontal at the time of deposition. Reversals of magnetization occur in many of the formations, for instance in the Antrim Plateau Basalts, and in these cases the two alternative directions are given. For those rock formations in which reversals have not so far been detected, the north-seeking directions are given. Only the southern hemisphere poles are quoted in the table and there are, of course, antipoles to these in the northern hemisphere.

Stereographic projections are used throughout. On these the mean pole position is indicated by a dot and is surrounded by an error area within which the pole lay at $P = 0{\cdot}95$. The area of error is circular when the sampling region was near the ancient pole, for example in the Permian volcanics, but is oval when the sampling region lay near the ancient equator. For a given sample size the former is eight times the area of the latter so that accurate pole determinations from rock formations deposited in high latitude require a comparatively large number of specimens.

Table of rock formations (all angles are in degrees).

No.	Formation	Geological age	Mean direction of magnetisation			Sampling Area		Pole position			
			D	I	α	Lat.	Long.	Lat.	Long.	$d\psi$	$d\chi$
14	Newer volcanics of Victoria	Pliocene, Pleistocene and Recent	3·4 183·4	−59·8 +59·8	4·8	38·0S	143·5E	86·3S	102·1E	5·5	7·2
13	Older volcanics of Victoria	Lower Tertiary, probably Eocene	17·0 197·0	−72·9 +72·9	6·8	38·0S	145·5E	66·8S	122·7E	10·5	12·1
12	Dolerite sills of Tasmania	Mesozoic, probably Jurassic	325·0	−85·0	3·5	42S	147E	50S	157E	6·8	6·8
11	Brisbane tuff	Triassic, probably lower Triassic	35 215	−83 +83	—	27·5S	153·5E	39S	143E	—	—
10	Volcanics of Illawarra coast	Permian, Upper Marine Series	67	+81	11	35S	151E	27S	169E	21	21
9	Volcanics of Hunter Valley	Permian, Lower Marine Series	110	+80	—	33S	151E	38S	174E	—	—
8b	Kuttung red varvoid sediments	Upper Carboniferous	270 90	−84 +84	6	33S	151E	32S	165E	12	12
8a	Kuttung Lavas	Upper Carboniferous	5	−85	8	33S	151E	43S	150E	16	16
7	Ainslie volcanics	Devonian, probably lower Devonian	17	−30	12	35S	149E	66S	12E	7	13
6	Mugga phorphyry	Upper Silurian	26	−30	22	35S	149E	60S	23E	14	24
5	Elder Mountain sandstone	Middle Cambrian	231	−15	10	16S	126E	34S	8E	5	10
4	Antrim plateau basalts	Lower Cambrian	53 233	− 2 + 2	12	16S	126E	36S	26E	6	12
3	Buldiva quartzite	Top of Upper Proterozoic	243	+38	12	14S	132E	30S	59E	8	14
2	Nullagine lavas	Upper Proterozoic	143	+64	8	21S	120E	51S	162E	10	13
1	Edith River volcanics	Lower part of Upper Proterozoic	90	+48	18	13S	132E	6S	166W	15	24

D is the declination reckoned east of geographic north; I is the inclination considered positive when below the bedding plane; α is the semi-angle of the cone of confidence around the mean direction at $P = 0{\cdot}95$ (Fisher 1953, Watson & Irving 1957); $d\psi$ and $d\chi$ are the semi-axes of the oval error area round the pole position at the same probability, $d\psi$ is the error in co-latitude (Creer, Irving & Runcorn 1957). In formation No. 12 reversals have been recorded, but they are of restricted occurrence.

2. Polar movement since the Carboniferous

The pole during the Upper Carboniferous (Nos. 8a and 8b) determined from the sediments and lavas of the Kuttung Series of the Hunter River valley lay in what is nowadays the Tasman Sea (Figure 2). In Permian times (Nos. 9 and 10) it lay a little to the north and east. It is interesting that this result confirms an early measurement by Mercanton (1926) who recorded an almost vertical magnetization in a single specimen from these lavas. By Mesozoic times the pole passes

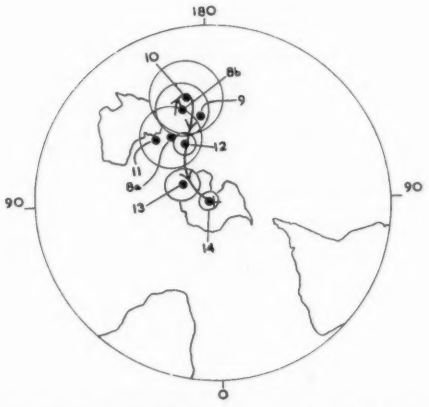

FIG. 2.—*Polar movement since the Carboniferous.* The poles are numbered as in the Table. The southern hemisphere poles are plotted, the antipoles being given in Figure 5. The approximate pole path is indicated in thick line (Polar projection).

into the Southern Ocean (Nos. 11 and 12) reaching the coast of Antarctica in the Lower Tertiary (No. 13). Since then it has coincided with the present South geographic pole (No. 14). The Northern hemisphere polar path for this time passes from North-west Africa in a broad sweep through the North Atlantic to the North geographic pole (Figure 5).

There is a suggestion in Figure 2 that the Permian pole is in advance of the Carboniferous one so that the direction of polar movement may have reversed during this time. However, the pole errors are large and verification by future measurements is necessary, but it is of interest that in the determinations from North America and Europe the Carboniferous pole positions also appear to be somewhat in advance of the Permian poles (see Figure 5, and also a review by Irving (1957b)).

3. Polar movement prior to the Carboniferous

For earlier times the pole path is sketched only in preliminary fashion (Figure 3). Three rock formations within the Upper Proterozoic of Australia (Adelaidian System) have been studied. The oldest are the Edith River Volcanics, a formation within the Catherine River Group, which belongs to the lower part of the Upper Proterozoic (Noakes 1956); the pole lay in the mid-Pacific (No. 1). The Nullagine lavas are part of a sequence which contains evidence of glaciation and for this reason are correlated tentatively with the Sturtian stage of the Adelaidian System (David 1950, p. 72), that is, about the middle of the Upper Proterozoic; the pole

bedding plane, α is the semi-angle of the cone of confidence about the mean pole position at the same probability, $d\psi$ is the error in co-latitude (Creer, Irving & Runcorn 1957). In formation No. 12 reversals have been recorded, but they are of restricted occurrence.

determined from these lavas lay near New Zealand (No. 2). The relative stratigraphic positions of the Edith River Volcanics and these Nullagine lavas is uncertain and it is assumed here that the latter are somewhat younger as seems most likely from the glaciation evidence. The Buldiva quartzite, which contains fossils of primitive life, is the youngest Proterozoic formation studied. It is placed at the very top of the Upper Proterozoic and is sometimes referred to as sub-Cambrian (Noakes 1956). The pole (No. 3) lay near Mauritius in the South Indian Ocean.

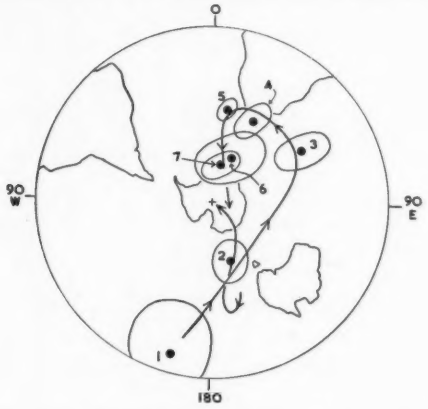

FIG. 3.—*Polar movement during and since the Upper Proterozoic.* The poles are numbered as in the Table. The pole path sketched is necessarily conjectural. The path of polar movement since Carboniferous times is added in thinner line for comparison. The possible linkage of these two through Antarctica is indicated (Polar projection).

The Upper Proterozoic appears to have been a period of very considerable polar wandering as the palaeomagnetic data from the Northern hemisphere also indicates (Du Bois 1955, Creer & others 1957).

The pole during Lower and Middle Cambrian times lay near South Africa (Nos. 4 and 5). During the late Silurian and early Devonian it was situated in a similar longitude but well south in the Southern Ocean.

Because of the occurrence of reversals of magnetization the palaeomagnetic method, although it determines the position of the pole, does not give its polarity. Clearly, unless an approximately continuous pole sequence is obtained, as for instance in Figure 2, an unambiguous path cannot be traced. In the Australian results there are very great discontinuities in the Upper Proterozoic and especially in the Upper Palaeozoic between the Lower Devonian and the Upper Carboniferous, and ambiguities of this type consequently arise. Clearly, the curve given in Figure 3 is only one of many possible polar paths (it is in fact the simplest possible path) and further sampling will be necessary before these uncertainties are settled.

4. Palaeoclimatic aspects

There are theoretical reasons connected with the origin of the geomagnetic field for supposing that the average geocentric dipole will always be directed along the Earth's axis of rotation (Runcorn 1954, Creer & others 1957). Palaeomagnetic evidence from the Upper Cenozoic in both hemispheres has shown that this has been the case during the last 20 million years (Hospers 1955, Irving & Green 1957). It is therefore possible that the pole positions given in this paper may also be the

positions of the geographic poles relative to Australia. If this is so the palaeoclimatic conditions in Australia should be broadly consistent with the geomagnetic latitude deduced from the palaeomagnetic data.

The variations in geomagnetic latitude for South-eastern Australia are given in Figure 4. During the Upper Proterozoic, and again in the late Palaeozoic and Mesozoic, Australia was in a high geomagnetic latitude and it is at these times that the geological evidence indicates a cold climate. The tillites and varved sediments

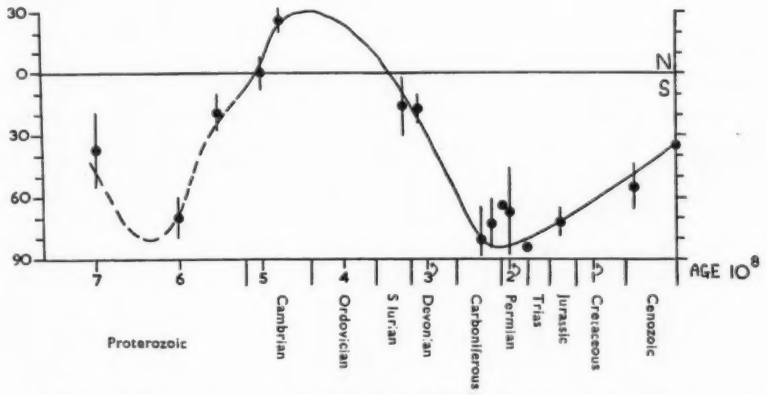

FIG. 4.—*The variations in geomagnetic latitude in South-eastern Australia.* These curves are calculated for a central point in South-eastern Australia (35S, 145E). The errors are obtained from the errors in the determinations of the positions of the ancient poles and represent the limits within which the latitude lay at $P = 0{\cdot}95$. The curve is derived from Figures 2 and 3.

of the Adelaidian System and the outwash gravels of the Nullagine rocks provide evidence of extensive glaciation in the earlier period (David 1950, p. 79). Similar deposits in the Kuttung Series and its equivalents elsewhere in Australia indicate a glaciation in the Upper Carboniferous (David 1950, p. 326). Evidence of glaciation is again found in the Permian (David 1950, pp. 341, 397), and the deposits of the Triassic, Jurassic and Cretaceous are generally regarded as having been laid down in cold to temperate conditions (David 1950, pp. 440, 476, 515). The fossil trees during these periods have pronounced growth rings suggesting the well-marked seasonal changes characteristic of higher latitudes. The great saurian reptiles are absent and the invertebrate marine faunas have a cold water aspect (see, for instance, Whitehouse 1926). The indications of warm climate which are widespread in other parts of the world at these times, for example in the Mediterranean region, are absent, and this would seem to imply that Australia was in high or intermediate latitudes just as the palaeomagnetic results do. It may also be noted that from the Proterozoic to Devonian the geomagnetic latitudes are low, and during these times no glacial formations are known in Australia, whereas red beds, dolomites, thick limestones with algal, Archaeocyathid and coral reefs are common.

Because of exceptional meteorological conditions in the past the palaeoclimatic evidence may not always give a close indication of latitude. Nevertheless the broad correspondence between this evidence and the palaeomagnetic latitude, in particular the coincidence of high geomagnetic latitude with glaciation, suggest that on average the geomagnetic and geographic poles have always been coupled together,

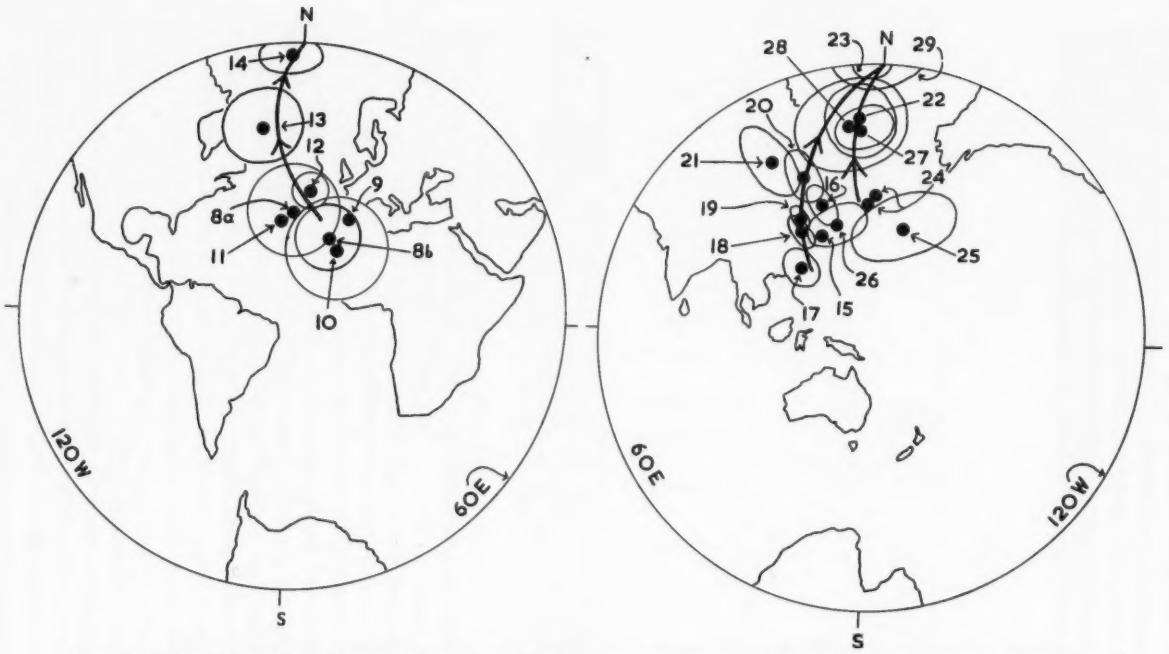

FIG. 5.—*Pole positions obtained from rock formations of Carboniferous and later ages in North America, Europe and Australia (equatorial projection).* The large discrepancy between equivalent results from Australia and those from northern continents is illustrated here. The individual poles for Australia are numbered as in the Table. The North American poles are numbered as follows:

15. Mississipian (Howell & Martinez 1957)
16. Pennsylvanian (Runcorn 1956)
17. Permian (Runcorn 1956)
18. Permian (Doell 1955)
19. Permian (Graham 1955)
20. Triassic (Runcorn 1956)
21. Triassic (Du Bois *et. al.* 1957).
22. Cretaceous (Runcorn 1956)
23. Upper Tertiary (Campbell & Runcorn 1956)

The European poles have these numbers:

24. Lower Carboniferous (Belshe 1955)
25. Permian (Creer & others 1957)
26. Triassic (Clegg, Almond & Stubbs 1954)
27. Eocene (Hospers 1955)
28. Oligocene (Roche 1950)
29. Miocene (Hospers 1955).

and that so far as Australia is concerned the great Proterozoic and late Palaeozoic glaciations may be reasonably explained by movement of the geographic poles relative to Australia. The ambiguity mentioned at the end of the previous section does not alter this argument. It should be noticed from Figure 2 that this explanation for glaciations in Australia would not provide for the late Palaeozoic glaciations of South America, South and Central Africa and India, assuming that the present continental positions have always been maintained.

5. Comparison with palaeomagnetic results from elsewhere

Except in later Cenozoic times the poles determined from the Australian data do not coincide with equivalent results from Europe, North America, Africa and India. The contrast is greatest in the Upper Palaeozoic and early Mesozoic where the discrepancies between pole determinations from rock formations of similar age is between 50° and 90° of arc, and are far greater than any experimental error (Figure 5). It would seem very difficult to explain these discrepancies without entertaining the possibility that in the past Australia has moved as a whole relative to the Northern continents.

The polar paths for Carboniferous and later times obtained from North America, Europe and Australia have an approximately similar form. They are roughly meridional in direction, begin in similar latitudes, and converge on the present pole in later Tertiary times. Their different longitudinal positions could be explained by supposing that relative movements have occurred between these continents since the Carboniferous, movements which possibly extended into the Tertiary (Irving & Green, 1957). However, the form of the polar path from Australia prior to the Carboniferous is so different from that obtained from the Northern continents (Creer & others 1957) that the supposition of post-Carboniferous continental drift *only* is inadequate and it is necessary to suppose that relative movements also occurred prior to the Carboniferous. *If* the discrepancies between equivalent pole results from different continents are to be explained by continental drift then there seems to be no reason to regard it as a unique process occurring only at one stage of geological history; in fact there are suggestions that it was also operative in earlier epochs.

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The European poles have these numbers: 18. Permian (Doell 1955) 19. Permian (Graham 1955) 20. Triassic (Runcorn 1956) 27. Eocene (Hospers 1955) 28. Oligocene (Roche 1950) 29. Miocene (Hospers 1955).

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A Seismic Refraction-Shooting Survey off the North Coast of Cornwall

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Summary

A seismic refraction-shooting survey has been carried out along four bearings from a hydrophone laid on the sea bed near Perranporth, Cornwall. The structure and acoustic properties of the sea bed have been deduced, and the attenuation of ground-waves with range has been measured both at this site and with a hydrophone suspended in a well nearby. The results obtained suggest that the sea bed in this area consists of three layers; a sand or sediment layer 80–100 ft thick with velocity 4950–5500 ft/s, a high velocity rock layer 1300 ft thick with velocity 14 250 ft/s, and a very thick high velocity layer with velocity 18 500 ft/s. The 14 000 ft/s layer is correlated in velocity with schists obtained from quarries near Perranporth, which suggests that this layer is a northerly extension of the Palaeozoic rocks of Devon and Cornwall. The attenuation of the ground-waves was found to be approximately 12 dB per double distance. A comparison of the results with the existing theories of the propagation of explosive sounds over elastic and fluid sea beds is made, and some recommendations for further research proposed.

1. Introduction

This paper describes the results obtained during a refraction-shooting survey carried out near Perranporth, Cornwall, in July, 1956. The survey was undertaken in order to determine the geological structure of the sea bed in this area, and the seismic method used was essentially the same as that used on land. The theory of the seismic refraction method is described by Bullard, Gaskell, Harland & Kerr-Grant (1940), Heiland (1946), Leet (1938) and Willmore (1949).

The possibility of using hydrophones rather than geophones as receivers for seismic refraction-shooting at sea has been described by Bullard & Gaskell (1941) and Hill (1952).

Refraction-shooting profiles were determined along bearings 226°T, 270°T, 316°T and 000°T, from a hydrophone laid on the sea bed in a depth of 17 fathoms and 2 miles off shore. 1 lb T.N.T. charges were fired on the sea bed along each bearing at intervals of one-eighth of a mile to a range of 2 miles, quarter-mile intervals to a range of 5 miles, and half-mile intervals to a maximum range of 10 miles. During the runs along bearings 316°T and 000°T, the ground-wave signals were also

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received on a hydrophone suspended 400 ft down a well located approximately quarter mile south-east of Cligga Head. The nominal positions occupied by the firing ship during the experiments are shown in Figure 1.

Navigation was carried out using the Decca Navigator and by utilising the green and purple lanes of the south-west British Decca Chain (which give almost right-angled intersections in this area) very satisfactory accuracy was achieved.

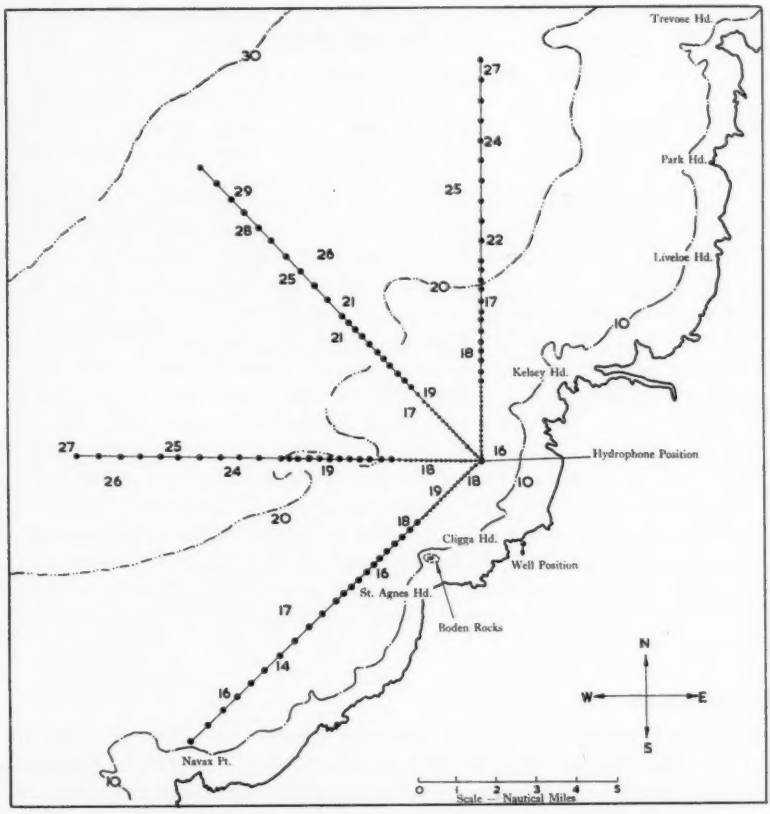

FIG. 1.—*Chart showing the four refraction-shooting runs.*

A comparison of the Decca positions and the positions calculated from the water-wave travel-times indicated an accuracy of 100–150 yd. The firing ship gave thirty seconds warning by radio before firing each charge, and the signal from the explosion, received on an accelerometer clamped to the ship's hull, was used to give the time-of-firing by breaking a C.W. radio transmission.

During the four runs, continuous echo-soundings were also made in order to determine accurate bottom profiles. The profiles computed from the echo-soundings and corrected for tidal height variations using Admiralty Tide Tables are shown in Figure 2. Bottom samples were also taken at 2-mile intervals along each run, using a conical dredge. These samples were invariably found to consist of a mixture of fine sand, small stones and broken shells.

The ground-wave measuring equipment used during the survey is shown in the block schematic diagram Figure 3.

2. The analysis of the seismic results

The main purpose of the survey was to determine the depths to, and velocities of, compressional waves in the different layers of strata of the sea bed, by measur-

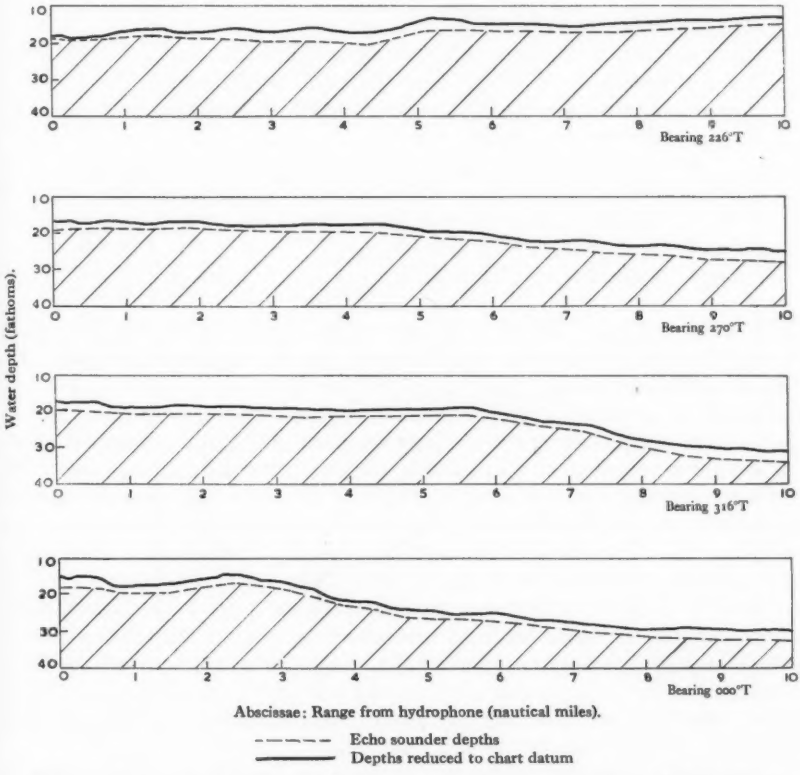

FIG. 2.—*Bottom profiles along four bearings from hydrophone position.* Hydrophone position Lat. 50° 22′ 20″ N, Long. 05° 11′ 49″ W.

ing the variations of ground-wave travel-time with range. As is usual with the analysis of seismic refraction-shooting results in an area which in depth is geologically unknown, it has been assumed that the compressional wave velocity increases discontinuously with depth. There is no evidence in this area to indicate that conditions exist which would produce a decrease of velocity with depth. By plotting the ground-wave travel-time against water-wave travel-time, the velocities of compressional waves in the various horizontally stratified layers of the sea bed and the depths of these layers can be determined from the following well-known equations:

$$t_{GW1} = \frac{c}{c_1} t_{WW},$$

$$t_{GW2} = \frac{c}{c_2} t_{WW} + \frac{2h_1}{c_2}\left(\frac{c_2^2}{c_1^2} - 1\right)^{\frac{1}{2}}$$

$$t_{GW3} = \frac{c}{c_3} t_{WW} + \frac{2h_1}{c_3}\left(\frac{c_3^2}{c_1^2} - 1\right)^{\frac{1}{2}} + \frac{2h_2}{c_3}\left(\frac{c_3^2}{c_2^2} - 1\right)^{\frac{1}{2}}$$

$$c_3 > c_2 > c_1 > c$$

where:

t_{WW} = travel-time of water-wave.
t_{GW1} = travel-time of ground-wave along c_1 interface.
t_{GW2} = travel-time of ground-wave refracted at c_2 interface.
t_{GW3} = travel-time of ground-wave refracted at c_3 interface.
c = velocity of sound in water.
c_1 = velocity of compressional waves in the 1st sea bed layer.
c_2 = velocity of compressional waves in the 2nd sea bed layer.
c_3 = velocity of compressional waves in the 3rd sea bed layer.
h_1 = depth of layer with velocity c_1.
h_2 = depth of layer with velocity c_2.

The three equations above represent straight lines, the gradients of which give the velocities in the layers, and the intercepts the depths to the layers. The theory of the interpretation of seismic travel-time curves in horizontal structures is described by Lehmann (1937), Muskat (1933) and Slichter (1932).

It frequently occurs that the line of lowest velocity on the time-distance diagram, for which there are ground-wave arrivals, does not pass through the origin. For the four refraction-shooting profiles described in this paper this occurred in every case, and the line of lowest velocity gave a positive intercept on the ground-wave travel-time axis. The positive intercept indicates the existence of a low-velocity sand or sediment layer overlying the first layer from which ground-wave arrivals were obtained. This layer probably represents Recent deposits but unfortunately it is impossible to determine its depth without a knowledge of the velocity of compressional waves in the layer. Therefore in this paper it has been assumed that the velocity lies between 1·0 and 1·1 times the velocity of sound in water. These velocity values represent the limits found from many measurements made on ocean sediments in America and the British Isles.

In reading the beginning of the ground-wave, the "earliest" definite indication of a disturbance of the ground-wave trace was taken. The time difference between the time-of-firing signal and the beginning of the disturbance gave the ground-wave travel-time. Due to the fact that the charges actually exploded slightly before the time-of-firing signal was transmitted, it was necessary to apply a correction to the apparent travel-times of the water-wave and ground-wave signals. A plot of ground-wave travel-time against water-wave travel-time then enabled the structure of the sea-bed to be determined.

Unfortunately, it was not possible to fire any reverse shots during the survey and therefore the component of dip along the four bearings has not been established. However, the value for the velocity of compressional waves in the second rock layer on bearing 270°T was found to be 4 per cent greater than the corresponding values obtained along the three other bearings, and in the theoretical analysis which follows in a later section of this paper, an attempt is made to determine whether this high value is consistent with an upward slope of this layer of strata.

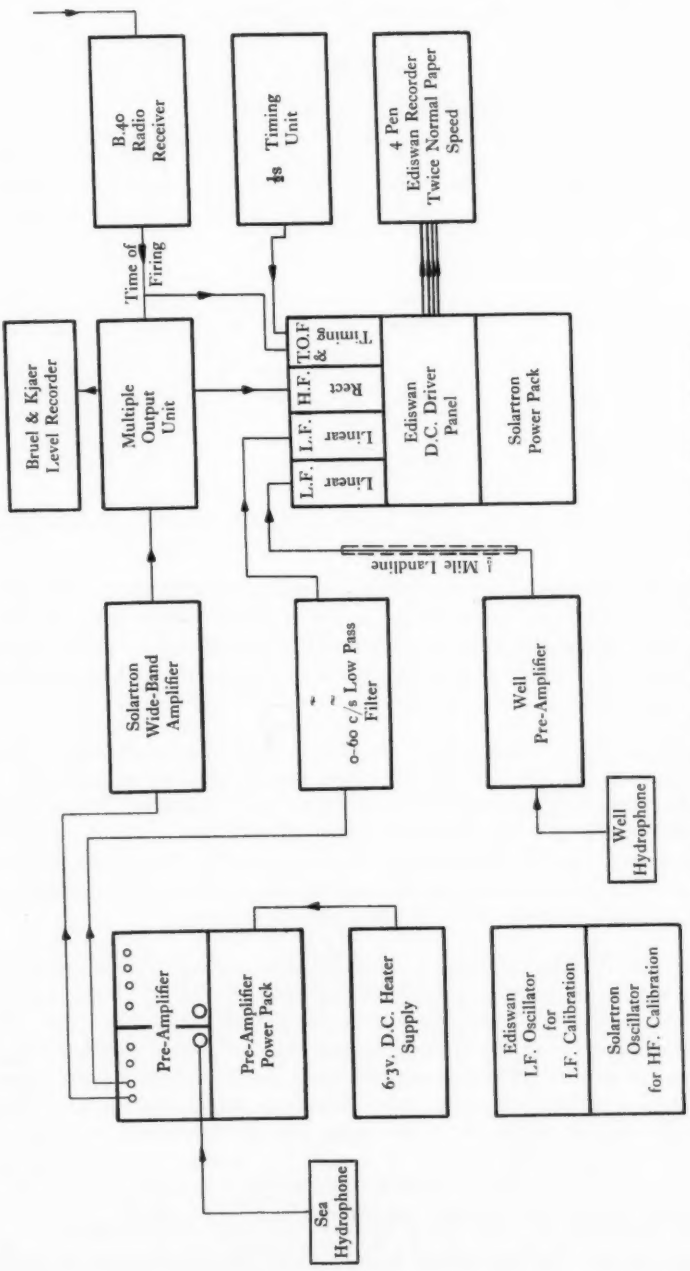

FIG. 3.—*Block schematic diagram of ground-wave measuring equipment.*

The travel-time curves derived from the experimental data are shown in Figures 4–9. The best straight lines were fitted to the points, using the method of least squares, and the final results of the analysis are given in Table 1.

Table 1

Layer Velocities and Depths

Bearing from Hydrophone	c_1/c Velocity Contrast in 1st layer	c_2 Velocity in 2nd layer ft/s	c_3 Velocity in 3rd layer ft/s	h_1 Depth to 1st layer ft	h_2 Depth to 2nd layer ft
226°T	1·0	10 700	18 300	110	650
	1·1	10 700	18 300	120	640
270°T	1·0	14 200	19 200	90	1280
	1·1	14 200	19 200	100	1280
316°T	1·0	15 700	18 500	130	1500
	1·1	15 700	18 500	140	1500
000°T	1·0	14 300	18 400	90	1250
	1·1	14 300	18 400	100	1250

It must be emphasised that the velocities shown in Table 1 are only apparent velocities, owing to the fact that reverse profiles were not determined. If the strata have a large dip, the velocities shown above may be considerably in error. Throughout the analysis, a velocity of 4950 ft/s for the velocity of sound in sea water has been assumed.

Bearing 226°*T.*—The travel-time curve for this bearing is shown in Figure 4. It can be seen that the curve shows rather complicated variations at the closer ranges, which probably indicate that the strata are not horizontal. This assumption is confirmed by the fact that Boden rocks (which rise 50–100 ft above sea level) are approximately on this bearing at a range of $2\frac{1}{2}$ miles. The deviations from a straight line of the points covering the range 0·8 to 4 miles may possibly indicate a slight upward slope of the 18 300 ft/s layer over this range.

Bearing 270°*T.* The travel-time curve for this bearing is shown in Figure 5. On the whole, the curve is in very good agreement with the simple ray theory, but the deviations from a straight line at the longer ranges are quite marked. A more detailed study of the curve indicates that the points covering the range 1·2 to 3·7 miles can be fitted to a velocity of 18 400 ft/s, and the points from 3·7 to 10 miles to a velocity of 19 150 ft/s. A possible explanation is that the third layer is horizontal out to a range of 3·7 miles, and thereafter has an upward slope of about 2°. A dip of this magnitude would indicate that outcropping of this layer should occur at a range of about 8 miles, and this is in good agreement with the indications of rock given on the Admiralty Chart of the area.

Bearing 316°*T.*—The travel-time curve for this bearing is shown in Figure 6. Although the curve tends to show a continuous change of gradient, the best straight lines were again fitted to the points. However, although the points covering

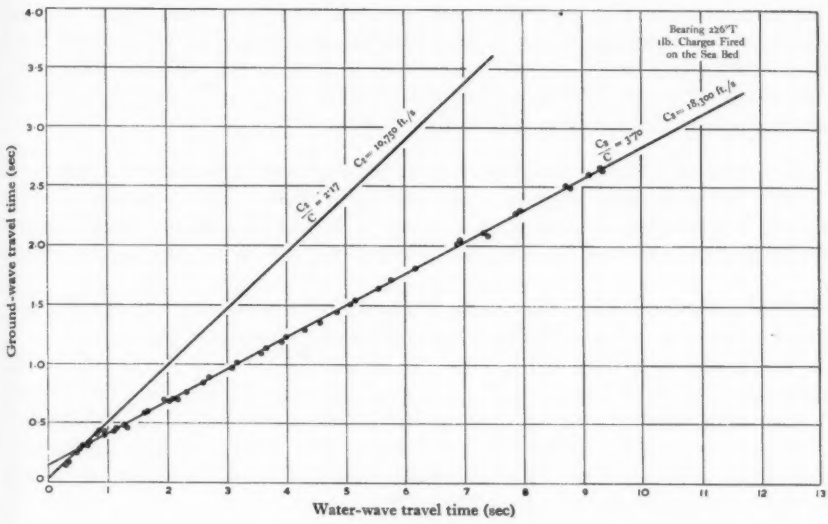

FIG. 4.—*Refraction-shooting profile along bearing* 226°T *from the hydrophone.* Hydrophone position Lat 50° 22′ 20″ N, Long. 05° 11′ 49″ W.

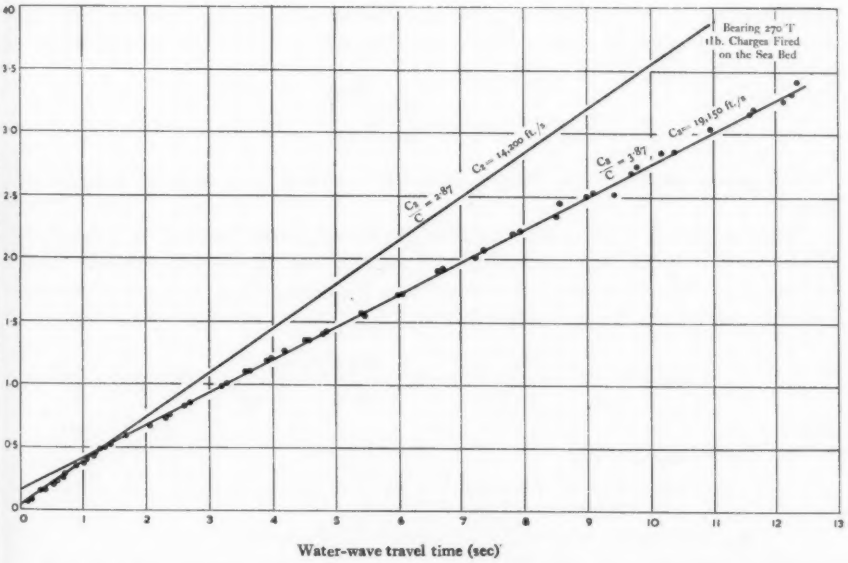

FIG. 5.—*Refraction-shooting profile along bearing* 270°T *from the hydrophone.* Hydrophone position Lat. 50° 22′ 20″ N, Long. 05° 11′ 49″ W.

the range 1·2 to 2 miles give a velocity of 18 500 ft/s the remaining points (although they give the same slope) are displaced upwards for ranges greater than 2 miles. This vertical displacement can be interpreted as an increase in depth of the third layer at this range, and is probably caused by faulting in the vicinity. These displacements of travel-time curves are not unusual and other cases have been reported by Day, Hill, Laughton & Swallow (1956).

Bearing 000°*T*.—The travel-time curve for this bearing is shown in Figure 7. The curve is again in good agreement with theory, but deviations from a straight line do occur at a range of 3 miles. It is possible that this slight discontinuity is associated with the decrease in water depth which occurs at this range.

Well hydrophone travel-time curves.—The travel-time curves for the ground-waves received on the well hydrophone are shown in Figures 8 and 9.

The gradients of these two curves give an average value of 18 150 ft/s for the velocity in the third layer, which is in good agreement with the other results.

The close range results for bearing 226°T shown in Table 1 are significantly lower than the results obtained on the three other bearings. If this low value is disregarded, the other results give average values of 14 700 ft/s and 18 500 ft/s for the velocities in the second and third layers.

3. The dip of the strata

The value of 19 200 ft/s for the velocity in the third layer for bearing 270°T is 4 per cent greater than the average value obtained from the other three bearings. The accuracy of the velocity measurements is known to be better than 2 per cent, and therefore it would appear to be worthwhile considering whether this high value is consistent with a dip of this layer. The standard formula for a two-layered bottom without dip is:

$$t_{GW2} = \frac{c}{c_2} t_{WW} + \frac{2h_1}{c_2}\left(\frac{c_2^2}{c_1^2} - 1\right)^{\frac{1}{2}}.$$

The variables used in the above equation have already been defined in a previous section.

If the sediment layer is disregarded, the Perranporth area can be treated as a two-layered case for dip considerations, and it can be shown that the above formula is modified to that given below when the second layer is assumed to slope upwards away from the receiving position.

$$t_{GW3} = \frac{c}{c_3}\frac{\cos(\theta_{23}+\alpha)}{\cos\theta_{23}} t_{WW} + \frac{2h_2\cos\alpha}{c_3}\left(\frac{c_3^2}{c_2^2} - 1\right)^{\frac{1}{2}}$$

where:

α = angle of dip.
c_2 = true velocity in 14 000 ft/s layer.
c_3 = true velocity in 18 000 ft/s layer.
θ_{23} = critical angle for c_2/c_3 interface = $\sin^{-1} c_2/c_3$.

Therefore, with an upward-sloping layer, the apparent velocity c_3' as given by the gradient of the refraction-shooting time-distance curves is:

$$c_3' = c_3\frac{\cos\theta_{23}}{\cos(\theta_{23}+\alpha)}.$$

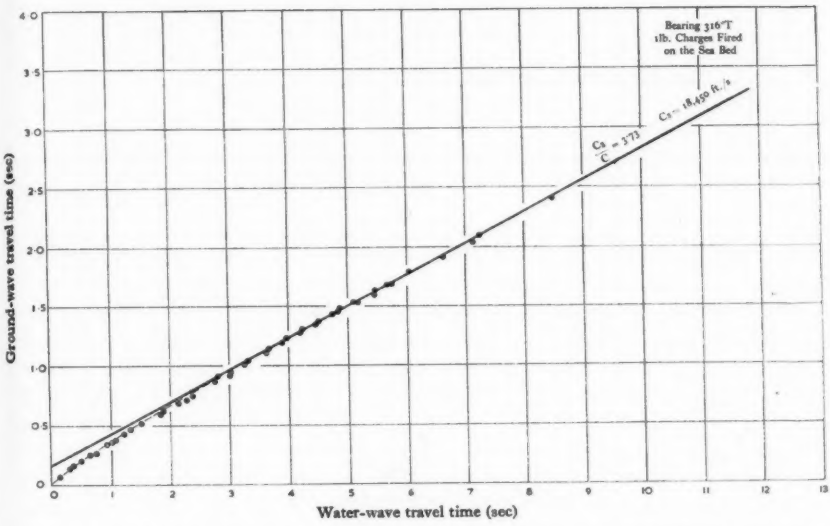

FIG. 6.—*Refraction-shooting profile along bearing* 316°T *from the hydrophone.* Hydrophone position Lat. 50° 22′ 20″ N, Long. 05°. 11′ 49″ W.

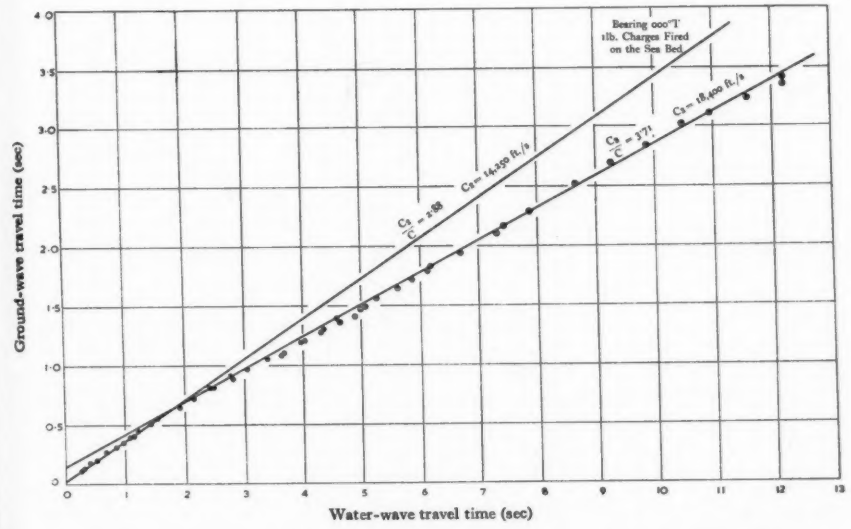

FIG. 7.—*Refraction-shooting profile along bearing* 000°T *from the hydrophone.* Hydrophone position Lat. 50° 22′ 20″ N, Long. 05° 11′ 49″ W.

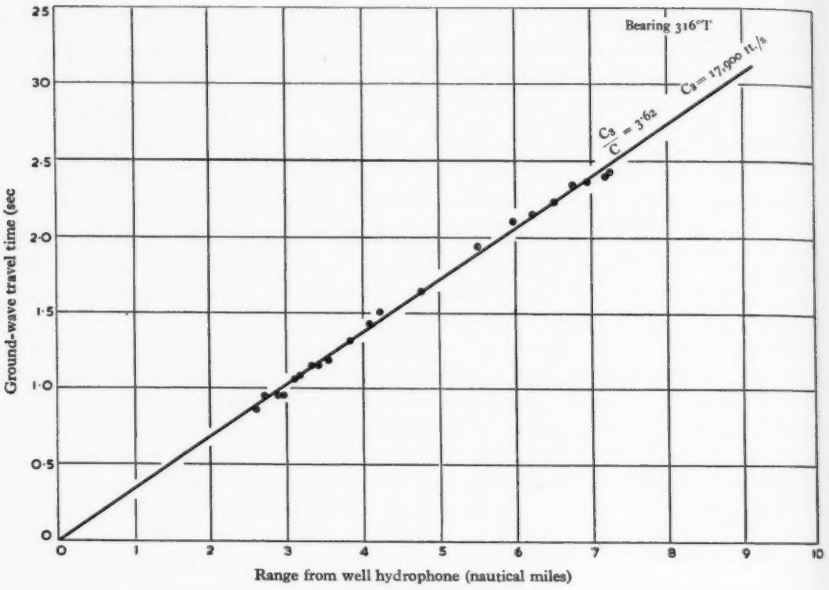

FIG. 8.—*Well refraction-shooting profile for run along bearing* 316°T *from sea hydrophone* Well hydrophone position Lat. 50° 20′ 13″ N, Long. 05° 10′ 18″ W. Well hydrophone 400 feet below ground level.

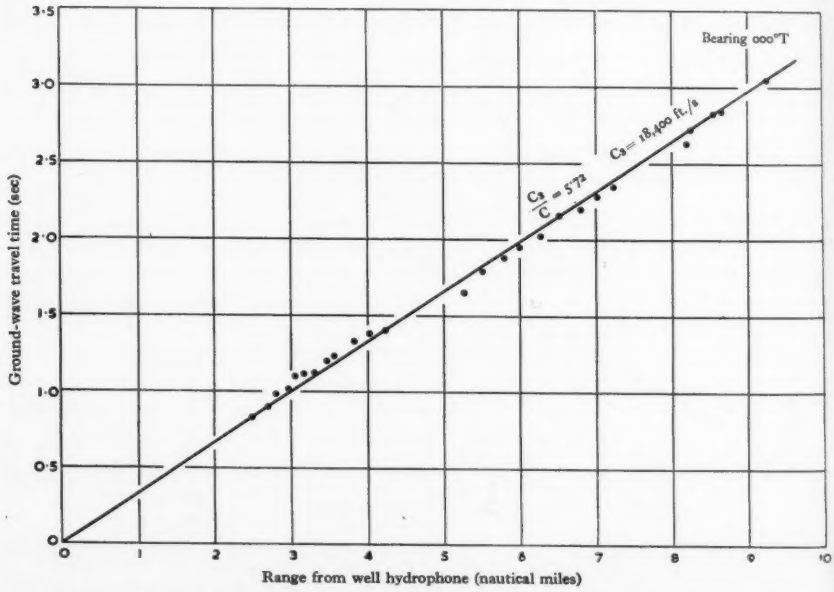

FIG. 9.—*Well refraction-shooting profile for run along bearing* 000°T *from sea hydrophone.* Well hydrophone position Lat. 50° 20′ 13″ N, Long. 05° 10′ 18″ W. Well hydrophone 400 feet below ground level.

For the particular case of the run along bearing 270°T, we assume that the apparent velocity c_3' of the third layer is 19 150 ft/s, and that the true velocity c_3 is 18 500 ft/s. Then by taking an average value of 14 250 ft/s for the velocity c_2 in the second layer, the angle of dip is given by:

$$\cos(39°42' + \alpha) = 0{\cdot}744$$
$$\alpha = 2°12'.$$

This value of 2°12′ is not unreasonable but unfortunately it cannot be checked owing to the fact that no reverse shots were fired during the experiments.

4. Compressional wave velocities in rocks from neighbouring land areas

To aid the interpretation of the velocities found from the refraction-shooting data, the velocities of compressional waves in three rock samples obtained from land areas close to Perranporth were measured in the laboratory of the Department of Geodesy and Geophysics at Cambridge. The apparatus used for these velocity measurements has been described in the paper by Day and others (1956). Approximately ten rock samples were sent to Cambridge, but most of these were found to be rather badly weathered, and only three were considered to be representative of the bulk of the country rocks in this area. The velocity measurements were all made at atmospheric pressure and in order to investigate the change of velocity with water-content, the rocks were wetted by evacuating air for several hours and then allowing water to enter the pore spaces. Table 2 below shows the velocities obtained for the three good specimens in the wet and dry condition.

Table 2

Velocities in schists from the north coast of Cornwall

Specimen	Thickness cm	Velocity dry ft/s	Velocity wet ft/s	Remarks
1	2·68	17 100	15 400	
	4·64	12 800	12 800	
	6·04	13 500	14 400	
	1·06	9 800	9 800	Perpendicular to lamination
2	4·30	13 100	13 100	
	4·89	15 100	15 800	
	1·08	9 800	9 800	Perpendicular to lamination
3	3 80	15 700	15 100	
	4 65	13 100	13 100	

The figures given in Table 2 indicate that there is a rather wide divergence between the three directions of measurement. In particular specimens 1 and 2 gave very low values when the measurements were made in the direction perpendicular to the laminations. The discrepancy between the measurements probably lies in the fact that the specimens were not pressurized. There is normally a rapid change of velocity with initial pressure and therefore the *in situ* velocity in these rocks is probably less sensitive to the direction of propagation than the laboratory

measurements indicate. Therefore, from the point of view of propagation of refracted waves, a value near the upper end of the range has been taken as the most appropriate.

In order to obtain the best average value for the velocity of compressional waves in these three rock samples, the results shown above, excluding the two low values measured perpendicular to the laminations, were averaged and the standard deviation calculated. The final results are as follows:

Mean velocity (dry) = 14 300 ± 600 ft/s.
Mean velocity (wet) = 14 200 ± 400 ft/s.

Although the accuracy of the above figures is not very great, the mean values are in very good agreement with the value of 14 250 ft/s obtained from the refraction-shooting data.

Unfortunately, no specimens of rock corresponding to the 18 000 ft/s layer were found in the cliffs and quarries near Perranporth.

5. Ground-wave frequencies

A full analysis of the ground-wave frequencies and the dispersion of the received signals has not been carried out, but from the results obtained it appeared that the low frequency component preceding the water-wave arrival had a frequency of 10·2–11·5 c/s with invariably some third harmonic also present. Two typical ground-wave signals for shots fired on the sea bed at ranges of 0·7 and 1·5 miles from the hydrophone are shown in Figure 10.

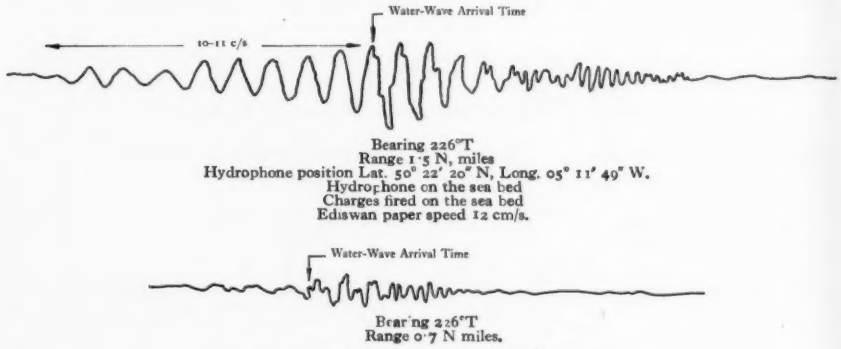
Water-Wave Arrival Time
10–11 c/s
Bearing 226°T
Range 1·5 N, miles
Hydrophone position Lat. 50° 22′ 20″ N, Long. 05° 11′ 49″ W.
Hydrophone on the sea bed
Charges fired on the sea bed
Ediswan paper speed 12 cm/s.
Water-Wave Arrival Time
Bearing 226°T
Range 0·7 N miles.

FIG. 10.—*Two typical ground-wave records.*

The longer range shots showed considerable amplitude modulation of the 10 c/s wave (presumably caused by a multiplicity of arrivals), and the ground-wave lasted for a considerable number of seconds after the arrival of the water-wave. In most records, a high frequency arrival of 40–45 c/s also appeared after the water-wave arrival. This frequency was usually only present over part of the range, and tended to disappear at the long range firing positions. At the longest ranges, the ground-wave was a nearly sinusoidal modulated 10 c/s wave; all the higher frequency components having disappeared. No frequencies lower than 10 c/s were observed during the experiments.

During spring tide periods at Perranporth there is a 20 per cent change in water depth at the position where the hydrophone was laid for these experiments.

This change should cause a detectable variation in the ground-wave frequency; it would therefore be of great interest to fire charges at a fixed position throughout a complete tidal cycle in order to investigate the variation of ground-wave frequency with time.

6. Ground-wave amplitudes

Owing to the fact that the ground-wave consisted of a complicated modulated 10 c/s wave, some criterion had to be used in order to obtain consistent results for the variation of amplitude with range. In this paper, the variation in amplitude of the first modulation maximum after the first indication of the beginning of the ground-wave has been taken. This corresponds to the "first arrival" maximum amplitude, and not to the overall maximum amplitude of the wave. The variations in amplitude with range and bearing are shown in Figures 11 and 12.

In all cases, the points show a rather large scatter, but a mean line through them indicates that the attenuation is approximately 12 dB per double-distance along bearings 270°T, 316°T and 000°T, and 9–10 dB along bearing 226°T. Along bearings 226°T and 270°T the attenuation tends to increase with range, but for bearings 316°T and 000°T it remains reasonably constant at 12 dB per double-distance. A loss of 12 dB per double distance corresponds to an intensity-range law of r^{-2} which is in good agreement with the ground-wave attenuations quoted in Worzel & Ewing (1948).

7. Comparison of results with theory

The ground-wave results described in this paper indicate that the sea bed at Perranporth consists of a low velocity layer of sand or sediment 80–100 ft thick in which the velocity of compressional waves is probably 4950–5500 ft/s. Beneath this top layer of sediment are two high-velocity layers of rock. The layer directly beneath the unconsolidated sediment is approximately 1300 ft thick and the velocity of compressional waves in it is 14 250 ft/s. The third layer is probably very thick and the velocity of compressional waves in it is 18 500 ft/s.

Using the above data, and the existing theories of the propagation of explosive sounds over elastic and fluid sea beds, an attempt will now be made to predict the main characteristics of the ground-waves in the Perranporth area. The theory for a sea bed which can be treated either as a semi-infinite fluid or a number of fluid layers was fully developed by Pekeris (1948), and the case of a semi-infinite elastic sea bed by Press & Ewing (1948, 1950) and Tolstoy (1954). Unfortunately, none of these theories is directly applicable to the Perranporth case, where the sea bed consists of a thin layer (which to a first approximation can probably be treated as a fluid) overlying a semi-infinite elastic solid. However, even though the comparisons with theory are rather crude, they are certainly worthwhile making in order to indicate the modifications required in order to obtain better agreement between theory and experiment.

Pekeris (1948) shows that the predominant frequencies in the ground-wave signal correspond to the cut-off frequencies of the normal modes. In most practical cases, the lower modes are more strongly excited than the higher modes, and one would therefore expect the main ground-wave frequency to correspond to the cut-off frequency of the first mode. The formulae shown below for the cut-off

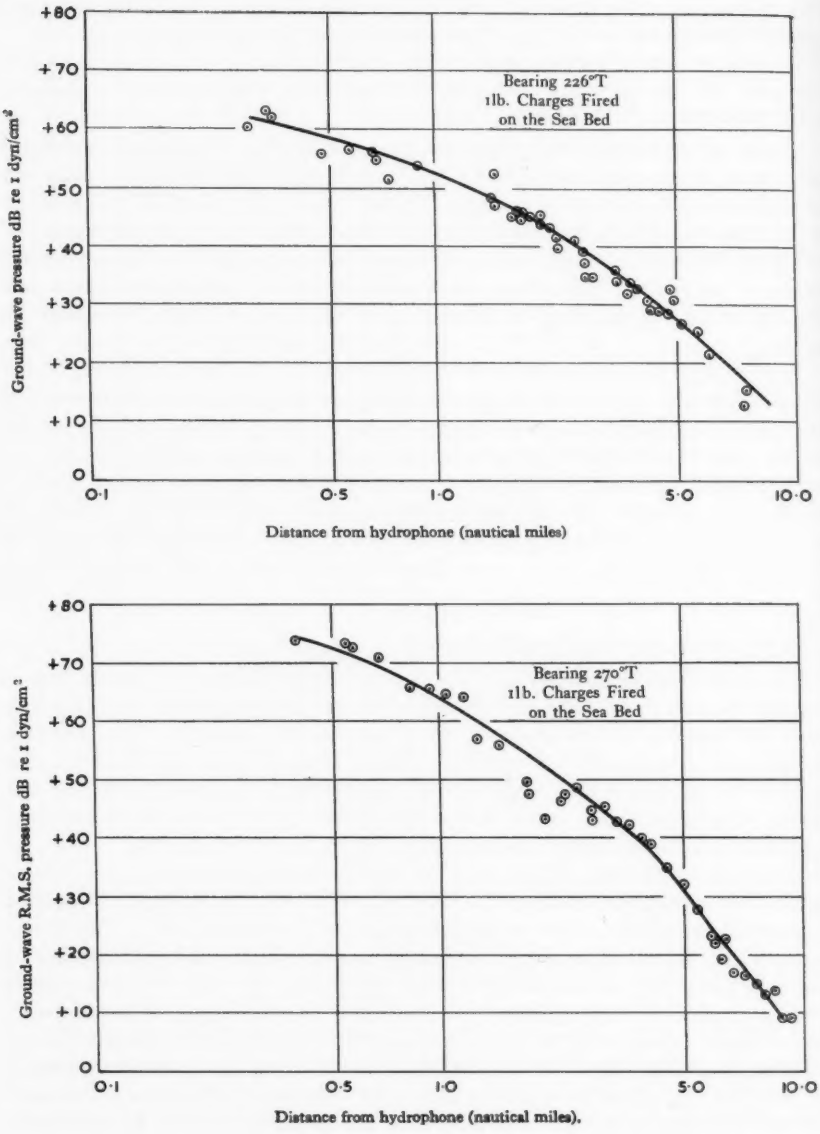

FIG. 11.—*Ground-wave transmission curves for bearings* 226°T *and* 270°T. Hydrophone on the sea bed at position Lat. 50° 22′ 20″ N, Long. 05° 11′ 49″ W.

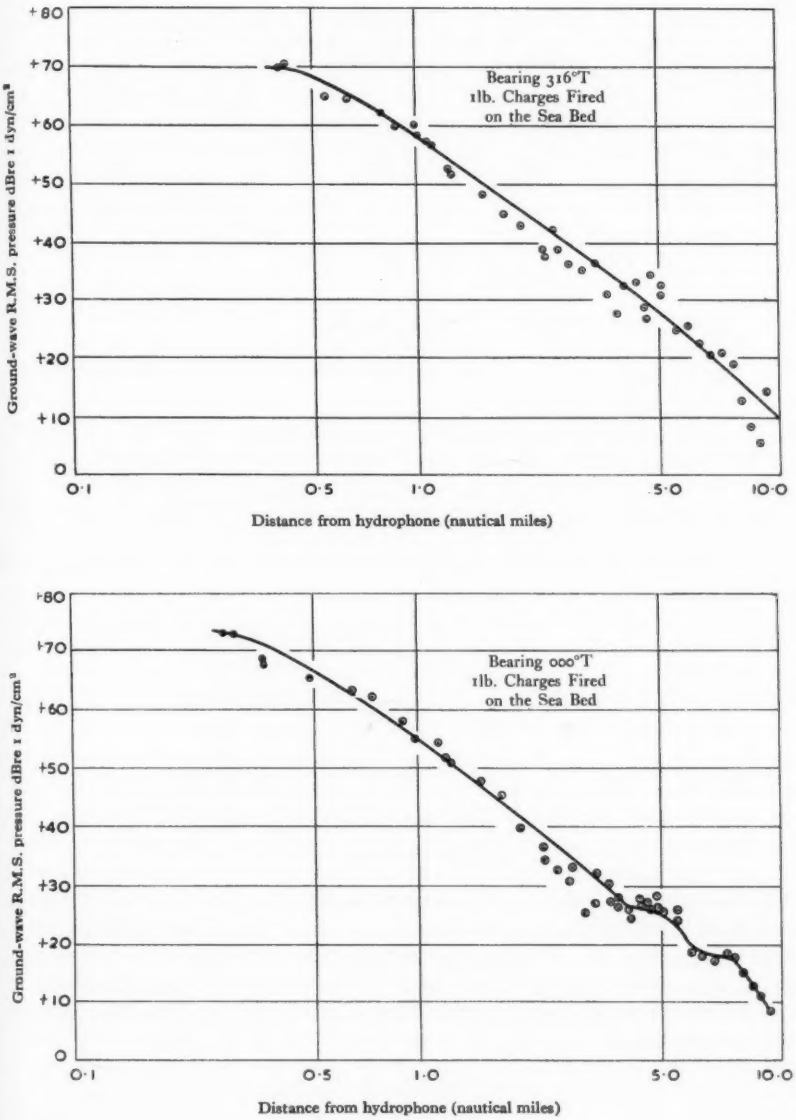

FIG. 12.—*Ground-wave transmission curves for bearings* 316°T *and* 000°T. Hydrophone on the sea bed at position Lat. 50° 22′ 20″ N, and Long. 05° 11′ 49″ W.

frequency of the nth mode for a three-layered fluid model will be used to determine the theoretical ground-wave frequencies to be expected in the Perranporth area.

$$f_c = \frac{(2n-1)c}{4Hu} \cdot \left(1 + \frac{\rho_1 c^2 v^2 h}{\rho_2 c_1^2 u^2 H}\right)^{-1} \text{ for } \frac{h}{H} < 0{\cdot}5$$

and

$$f_c = \frac{(2n-1)c_1}{4hv} \cdot \left(1 + \frac{\rho_1 H}{\rho_2 h}\right)^{-1} \quad \text{for } \frac{h}{H} > 2{\cdot}0$$

where

$$u^2 = 1 - \frac{c^2}{c_2^2} \quad \text{and} \quad v^2 = 1 - \frac{c_1^2}{c_2^2},$$

f_c = cut-off frequency of nth mode.
h = intermediate or sediment layer thickness.
H = depth of water.
ρ_1 = density of water.
ρ_2 = density of intermediate or sediment layer.
c = velocity of sound in water.
c_1 = velocity of compressional waves in sediment layer.
c_2 = velocity of compressional waves in underlying layer.

The following experimental data will be used to derive the 1st mode cut-off frequency from the above formulae:

H = 105 ft.
h = 110 ft.
c = 4950 ft/s.
c_1 = 5445 ft/s.
c_2 = 14 250 ft/s.
ρ_1 = 1·03 g/cm^3.
ρ_2 = 1·80 g/cm^3.

Using these figures, the cut-off frequency of the 1st mode is:

$$f_c = 8{\cdot}6 \text{ c/s for } h/H < 0{\cdot}5$$
$$f_c = 8{\cdot}7 \text{ c/s for } h/H > 2{\cdot}0.$$

The experimental cut-off frequency is 10·85 c/s and therefore the three-layered fluid model gives values which are too low. However, by reversing the process and putting the experimental cut-off value of 10·85 c/s into the above formulae, the depth of the sediment layer can be found and compared with the result obtained from the refraction-shooting data. If this is done, the following values are obtained:

$$h = 38{\cdot}0 \text{ ft for } h/H < 0{\cdot}5$$
$$h = 86{\cdot}5 \text{ ft for } h/H > 2{\cdot}0.$$

The use of the above formulae to determine the depth of the sediment layer is not strictly valid owing to the fact that at Perranporth h/H is 1·0. However, the average value of 57 ft for the sediment depth agrees reasonably well with the value given by the refraction-shooting data.

In order to see whether better agreement with the experimental results can be obtained by assuming a semi-infinite elastic sea bed, the 1st mode cut-off frequency

for this case will be calculated. The formula for the cut-off frequency for the nth mode over an elastic sea bed is as follows:

$$f_c = \frac{(2n-1)c}{4H}\left(1 - \frac{c^2}{\beta^2}\right)^{-1}$$

where β is the velocity of shear waves in the sea bed.

For the purposes of this calculation it will be assumed that the velocity in the sediment layer is so close to that in water that the depth of water H in the above formula can be taken as the true depth plus the depth of the sediment layer. If this assumption is made and the following experimental data used:

$$H = 105 + 110 = 215 \text{ ft},$$
$$c = 4950 \text{ ft/s},$$
$$c_2 = \sqrt{3}\,\beta,$$

the first mode cut-off frequency is given by:

$$f_c = 7{\cdot}2 \text{ c/s}.$$

The ratio $\sqrt{3}$ of compressional to shear-wave velocities in the sea bed used in the above formula is the value assuming a Poisson's ratio of 0·25. The cut-off frequency of 7·2 c/s using this value is low compared with the experimental value of 10·85 c/s. However, by reversing the procedure and using the experimental value of 10·85 c/s in the formula it is possible to determine the ratio of compressional to shear-wave velocities in the sea bed. If this is done a ratio of 2·4 is obtained, which is a reasonable value for the ratio of compressional to shear-wave velocities in the hard rocks near the earth's surface.

As a result of the calculations carried out above, a ground-wave of 7–9 c/s with second and third modes with frequencies of 21–27 c/s and 35–45 c/s would be expected to occur in the Perranporth area. These frequencies are in fair agreement with the experimental values of 10 c/s, 30 c/s and 40–45 c/s, but the discrepancies are certainly significant and are due to the shortcomings of the existing theories and the lack of accurate knowledge of the *in situ* acoustic properties of the sediment layer. Good agreement between theory and experiment will probably only be possible when the theoretical case of a thin fluid layer or a thin layer with very low rigidity (in which the velocity increases with depth) overlying a semi-infinite elastic solid is available. Many interesting cases of propagation in layered media are discussed in a recently published book by Ewing, Jardetzky & Press (1957).

With regard to the ground-wave variation of amplitude with range, it has been reported that an r^{-1} law or greater is to be expected theoretically for the fluid bottom case. The results reported in this paper show that an r^{-2} law applies in the Perranporth area, and this ground-wave intensity fall off of 12 dB per double-distance is in good agreement with ground-wave intensity losses quoted in Worzel & Ewing (1948).

From the foregoing qualitative analysis, it appears that neither the simple fluid nor the elastic sea bed theories can fully explain the ground-wave propagation at Perranporth. The sediment depth of 100 ft is sufficiently small to allow the first rock layer greatly to affect the cut-off frequencies, and therefore the predominant periods of the refracted waves. The high rate of attenuation of the ground-waves is probably due to the fact that these very low frequencies penetrate deep into the sea bed, and therefore suffer heavy shear-wave damping at the sediment-rock

interface. The higher frequencies would probably not suffer such heavy damping owing to their smaller depths of penetration, but would be attenuated due to scattering and absorption. If theoretical group-velocity curves could be computed using the Perranporth parameters reported in this paper, a much better insight into the mechanism of ground-wave propagation in coastal waters would probably be possible.

8. Conclusions

The structure of the sea bed near Perranporth, Cornwall, has been determined with reasonable accuracy as a result of the refraction-shooting experiments described in this paper. The top layer consists of a low-velocity sand or sediment layer approximately 80–100 ft thick, in which the velocity of compressional waves is probably 4950–5500 ft/s. Beneath this top layer of sediment there are at least two high-velocity layers of rock. The layer directly beneath the sediment is approximately 1300 ft thick and the velocity in this layer is 14 250 ft/s. The third layer is probably very thick and the velocity of compressional waves in it is 18 500 ft/s.

Refraction-shooting experiments carried out in the English Channel and described by Hill & King (1953) and Day & others (1956) indicated the existence of strata with velocity lying between 12 000 and 15 900 ft/s. These strata were found to be correlated in velocity and on stratigraphical grounds with the Palaeozoic terrain of Devon and Cornwall. The velocity of the first rock layer found by refraction-shooting at Perranporth also lies between the limits found for the Channel layer, and is also correlated in velocity with the rock samples from the north coast of Cornwall. This evidence would therefore suggest that the Palaeozoic rocks of Devon and Cornwall also form a continuous layer beneath the sea, north of the Cornish peninsula.

The predominant frequency of the ground-wave at Perranporth was found to be approximately 10 c/s and the intensity loss as a function of range was found to follow an r^{-2} law.

The experiments described in this paper have also shown that although the refraction-shooting method is very useful for determining the deep structure of the sea bed it gives very little information on the sediment layer, unless this happens to be very thick. The minimum range of 250 yd used in the experiments was not sufficiently small to enable the wave which had travelled through the sediment to be resolved, and therefore an accurate value for the velocity in this layer is still not known.

Fair agreement between the experimental results and the existing theories of the propagation of explosive sounds over fluid and elastic sea beds has been obtained, but accurate agreement will probably only be possible when new techniques have been developed, which will enable the vital parameters of the upper sediment layers of the sea bed to be determined.

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1957 *September.*

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A Modification of Lomnitz's Law of Creep in Rocks

Harold Jeffreys

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Summary

C. Lomnitz has shown that creep of rocks under sustained stress increases logarithmically with time. Data for the variation of latitude and seismic waves indicate that for most of the Earth's shell a term in t^α, with α about 0·17, is better. This rule is applied in the present paper to the subsidence of a harmonic surface inequality. The indications are that this type of creep will not account for isostatic adjustment within a geological period, and the presumption is that the adjustment is due to fracture or flow near the elastic limit.

In recent papers (Jeffreys 1957a, b) on tidal friction, I assume a law of elastic afterworking such that the displacement under constant stress P applied from $t = 0$ is

$$\epsilon = \frac{P}{\mu}\left\{1+\left(\frac{\tau'}{\tau}-1\right)\left(1-e^{-t/\tau'}\right)\right\}.$$

This has the properties that (1) the initial strain has the elastic value P/μ and (2) the strain under continued stress increases asymptotically to a new value $P\tau'/\tau\mu$. τ'/τ was somewhat arbitrarily taken as 1.1 (there being good evidence that it is less than 1.6). Under this law the lag of a harmonic vibration is

$$\eta = \tan^{-1}\gamma\tau' - \tan^{-1}\gamma\tau$$

where $2\pi/\gamma$ is the period. The damping of the variation of latitude indicates that for a period of 430 days η is about 1/40, (Jeffreys 1952, p. 247). This gives two solutions for τ and τ', namely

$$\tau = 17{\cdot}0\text{d}, \quad \tau' = 18{\cdot}7\text{d};$$
$$\tau = 251\text{d}, \quad \tau' = 276\text{d}.$$

These are referred to as the short and the long time scale.

On application to the rotations of the Moon and Mercury, I found that the short time scale gives enough tidal friction to account for the fact that they keep constant faces to their primaries. The long time scale did not give enough for Mercury and doubtfully gave enough for the Moon. Elasticoviscosity is the limiting case obtained by abandoning the hypothesis $\tau'/\tau = 1{\cdot}1$ and putting τ' infinite. It was found to be even more unsatisfactory.

As the law was assumed mainly for convenience without being qualitatively obviously wrong, the interpretation to be put on the results is simply that the lags

for periods of the order of 12 hours to a month are larger than the long time scale would imply but are consistent with the short time scale.

Many investigators of creep in solids have found that the strain increases with time like $\log t$ or $t^{1/3}$. Such rules have the obvious defect that they make the initial rate of strain infinite, and also, since the strain would ultimately increase indefinitely, they do not account at once for the fact that mountains and even buildings continue to stand.

C. Lomnitz (1956, 1957) proposes a simple modification which removes the former difficulty. According to him

$$\epsilon = \frac{P}{\mu}\{1+q\log(1+at)\}$$

where q and a are positive constants. In experiments on rocks he finds values for q from 0·001 to 0·010, and a of order 10^3 cycles/sec $\doteq 6\times 10^3$/sec. The stresses were less than $5\times 10^{-4}\mu$; and thus reached moderate fractions of the breaking stress. It is thus only from experiments at audible frequencies that a can be measured. At higher frequencies the lag is nearly $\frac{1}{2}\pi q$ irrespective of the period.

I examine the form of a unit plane pulse, which for a perfectly elastic solid can be represented by

$$H(t-x/\beta) = \tfrac{1}{2}+\frac{1}{\pi}\int_0^\infty \sin\kappa(\beta t-x)\frac{d\kappa}{\kappa}.$$

If there is a lag η independent of period this is modified to

$$\tfrac{1}{2}+\frac{1}{\pi}\int_0^\infty \sin\kappa(\beta t-cx)\exp(-\kappa sx)d\kappa/\kappa$$

where $c=\cos\frac{1}{2}\eta$, $s=\sin\frac{1}{2}\eta$; and this is

$$\tfrac{1}{2}+\frac{1}{\pi}\tan^{-1}\frac{\beta t-cx}{sx}.$$

This rises from $\frac{1}{4}$ to $\frac{3}{4}$ when $\beta t-cx$ rises from $-sx$ to $+sx$. Thus a pulse is blunted, the scale of the blunting (in time) being specified by $2sx/\beta$ or $2st$. Now if the lag was 1/40 it would imply that an S wave at 80° or so, having travelled for 20 minutes, would have its beginning spread over half a minute and be unreadable. The two data both concern the greater part of the Earth's shell and should be comparable, and to reconcile them it seems necessary that the lag for periods near 6sec should be about 1/15 of that for a period of 430d.

I therefore modify Lomnitz's rule to

$$\epsilon = \frac{P}{\mu}\left[1+\frac{q}{\alpha}\{(1+at)^\alpha-1\}\right]$$

which reduces to his when $\alpha\to 0$. This makes the lag (Jeffreys 1958)

$$\eta = \tfrac{1}{2}\pi\alpha\lg q\left(\frac{a}{\gamma}\right)^\alpha,$$

and the two data imply $\alpha = 0{\cdot}17$. This is about midway between the values 0 and 1/3.

The constant factor being taken to fit the lag for period 430 days, it is found that the lags for periods from 6 hours to about 50 days are greater than for the short time scale model previously used, and therefore are consistent with the data from tidal friction. From 6 hours to 10 days they are in the neighbourhood of 0·01, which is near Lomnitz's larger values (which were based on periods up to some weeks).

I have previously considered (1952, pp. 41, 107, 247) the hypothesis that the blunting of seismic pulses (mainly in the upper layers) is due to scattering, using a model due to Bridgman. The result gave an error function instead of the inverse tangent, which has the same general appearance; but there was no independent check, and it is a merit of the present rule that it accounts for the data simultaneously.

It is now possible to examine afresh the problem of the support of mountains. I take the simple case of harmonic loading of a plane surface, so that the load applied at time 0 is $P\cos\kappa x$. Since inertia can still be neglected the stresses will be proportional to those in an elastic solid, for which the depression is $1/2\mu\kappa$ times the load. The actual depression will be $\epsilon\cos\kappa x$ and the load at the original level of the surface $(P-g\rho\epsilon)\cos\kappa x$; and

$$\epsilon = \frac{1}{2\mu\kappa}\int_{\tau=0-}^{t}\left[1+\frac{q}{\alpha}[\{1+a(t-\tau)\}^{\alpha}-1]\right]d(P(\tau)-g\rho\epsilon(\tau))$$

$$= \frac{1}{2\mu\kappa}(P-g\rho\epsilon)+\frac{qa}{2\mu\kappa}\int_{0-}^{t}\{P(\tau)-g\rho\epsilon(\tau)\}\{1+a(t-\tau)\}^{\alpha-1}d\tau.$$

We are considering such times that $a(t-\tau) \gg 1$ over nearly the whole interval; hence we can replace $1+a(t-\tau)$ by $a(t-\tau)$. This is an integral equation of Abel's type for ϵ and can be solved by Goldstein's method. (H. and B. S. Jeffreys 1956, §12.14). Writing $\bar{\epsilon}$ for the operational form of ϵ and p for the Heaviside symbol we have

$$\bar{\epsilon} = \frac{1}{2\mu\kappa}(P-g\rho\bar{\epsilon})+\frac{q}{2\mu\kappa}\frac{a^{\alpha}}{p^{\alpha}}(\alpha-1)!(P-g\rho\bar{\epsilon})$$

and the surviving load at $x = 0$ is

$$P-g\rho\bar{\epsilon} = \frac{P}{1+(g\rho/2\mu\kappa)[1+qa^{\alpha}(\alpha-1)!/p^{\alpha}]}.$$

On interpretation this will ultimately tend to 0 like $t^{-\alpha}$. But if t is not too great we can expand in negative powers of p and interpret term by term, obtaining a useful convergent expression. Let us therefore consider ordinary values of the quantities; I take $\mu/\rho = 20\times10^{10}$ (cm/sec)^2; $2\pi/\kappa = 10^8$ cm. Then

$$\frac{g\rho}{2\mu\kappa} = 0{\cdot}04.$$

Also from the variation of latitude we know qa^{α}. If we take $t = 10^8$ years $= 3\times10^{15}$ sec,

$$q(\alpha-1)!(at)^{\alpha} \doteqdot 4.$$

Thus in a time of the order of a geological period the displacement due to later terms of the convergent expansion would be of the order of 16 per cent of the first. Thus linear creep is unlikely to account for considerable isostatic adjustment in a geolcgical period.

Since the stresses needed to support mountains are of the same order of magnitude as the elastic limits of rocks, the presumption is that the greater part of isostatic adjustment is by fracture or by flow near the elastic limit.

One conclusion to be drawn is that the hypothesis of elasticoviscosity in the Earth's shell (that is, rate of creep constant for constant stress) has fulfilled any useful purpose that it might ever have had, and no longer needs serious consideration. Its merits were that it provided qualitative explanations of the secular acceleration of the Moon, of the fact that the Moon and Mercury keep constant faces to their primaries, and of isostasy; and its mathematical application was not too difficult. It became clear in 1915 that if it accounted for the secular acceleration of the Moon it would prevent the existence of the 14-monthly variation of latitude, but it was found that turbulence in shallow seas could provide an explanation without requiring imperfection of elasticity in the body of the Earth. For a long time there was no positive geophysical evidence at all for such imperfection at small stresses. The 14-monthly variation of latitude is the phenomenon most sensitive to it, and at last it was found to show a measurable damping. But if this is attributed to elasticoviscocity the results still do not account for the rotation of Mercury and doubtfully do so for the Moon. Further, the explanation of isostasy breaks down because it predicts very different time-scales for the compensation of inequalities of different horizontal extents, contradicting the fact that uncompensated inequalities of all horizontal scales are observed. The only reason for continuing interest in elasticoviscosity has been the lack of a specific alternative. My model for elastic afterworking, with the short time scale, meets some of the difficulties, but not all; but it now appears that the modified Lomnitz law meets all the astronomical data. It apparently cannot explain isostatic compensation, which may, however, be attributed to fracture or flow near the elastic limit.

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XIth General Assembly of the International Union of Geodesy and Geophysics

Toronto 2nd–14th September, 1957

It is impossible to give any brief balanced account of the discussions at this Assembly, the proceedings of which will, in any case, appear fully in various other publications.

The editors have therefore chosen, from among the meetings they or their colleagues attended, those which seemed to them of particular interest, and reports of these meetings appear below.

Symposium on the Measurement of Gravity at Sea

The meeting of Section IV (Gravity) of the *International Association of Geodesy* on September 9th was devoted to a symposium on the measurement of gravity at sea.

Opening the symposium, *Mr B. C. Browne*, the President of the Section, pointed out that in the thirty years since Professor Vening-Meinesz developed his three-pendulum apparatus no alternative had been produced. Yet any equipment that could be used on a surface vessel would have great advantages, for the accuracy of submarine observations is limited by the precision with which the ship's position and velocity, which are required in correcting the observed apparent value of gravity, can be found. Furthermore, it would be far easier to get sea time for measurements on a surface ship than it is on a submarine. Another important convenience of a gravity meter would be that the results would be available almost immediately. To give some idea of the conditions under which a gravity meter should operate, Mr Browne said that the National Institute of Oceanography had found that under average summer conditions in the north-east Atlantic the motion of an instrument mounted near the centre of a ship would have an amplitude of approximately 1m and a mean period of around 6s. The corresponding r.m.s. vertical acceleration was 100,000 mgal and the second-order correction, 5000 mgal.

Dr A. Graf described the tests of his gravity meter which had been made in an American submarine in co-operation with Dr J. Worzel. The instrument is a spring balance instrument similar to the land gravity meter made by the Askania company. The motion of the beam is heavily damped and is recorded on moving paper so that it may be integrated over a suitable period to give the average apparent value of gravity. The apparatus is suspended in gimbals so adjusted that the effect of horizontal accelerations is just compensated by the inclination of the apparatus to the vertical. If the wave motion is small, the record shows a slow smooth variation of amplitude about 150 mgal, arising from imperfect depth keeping; in rougher seas the accelerations due to wave motion are superposed. In the Mediterranean, where the sea was quiet, the r.m.s. difference between the gravity meter and a

Vening-Meinesz pendulum apparatus was 1·2 mgal, in the Atlantic, with rough weather, it was 2·3 mgal with differences of up to 20 mgal, probably due to errors in depth keeping, and in the English Channel, where again the weather was quiet, the r.m.s. difference was 1·2 mgal. Dr Graf is now constructing an improved instrument.

It is necessary to have an instrument with a very large linear range for otherwise the average effect of the periodic vertical accelerations will not be zero. Dr Graf has checked the behaviour of his meter by giving it artificial vertical accelerations of up to 30,000 mgal and finds that the mean value is not more than 2 mgal in error. When used in a submarine, the lack of linearity of the instrument is no problem, but in a surface vessel the linearity must be very high and the second-order correction must be measured very accurately. It is not possible to do so accurately enough with a long period pendulum; alternatively a gyro-stabilized platform may be used or a gyroscope may be used to measure the fluctuations of the direction of the vertical. Dr Graf has made some experiments in a surface vessel on an inland sea.

Dr J. Worzel said that the Vening-Meinesz apparatus could be used with accelerations of up to 40,000 mgal. The periods of ocean waves were generally less than 10s and the amplitudes were sufficiently reduced at a submarine's cruising depth but, twice in the Pacific and once in the Atlantic, waves with periods of 17s had been encountered, the amplitudes being almost as great at maximum depth as on the surface. The problem of studying the behaviour of a gravity meter was far greater in a submarine than on land because stations could not be re-occupied exactly.

Dr Yu. D. Bulanzhe said that in Russia experiments were being made both with pendulums and with gravity meters. The pendulum apparatus has two sets of three pendulums so that two independent results can be obtained for each point. The horizontal and vertical accelerations are measured with pendulums having 40s periods. The gravity meter is of the Nørgaard type, heavily damped. During 1957, experiments were made on surface ships and submarines. The uncertainties of both the pendulum apparatus and the gravity meter on the surface ship were about three or four milligals when the horizontal accelerations were less than 25,000 mgal. For accelerations of 40,000 mgal the accuracy was less. In further developments it is intended to use a stabilized platform.

Sir Edward Bullard pointed out that although a gravity meter may be used on a stabilized platform, a pendulum apparatus cannot be.

Dr J. C. Harrison described submarine gravity measurements carried out by the University of California. One hundred and twenty-two observations had been made with a Vening-Meinesz apparatus and, in addition, tests of three La Coste sea gravity meters (see below) had been undertaken. Observations with two gravity meters (which were subject to the same horizontal accelerations and other perturbations) usually agreed to within 2½ mgal but, on occasion, gravity meter results differed by up to 17 mgal from pendulum observations. The pendulum apparatus and gravity meter (which incorporates an automatic computer that calculates the second-order correction) occupy about the same space in a submarine. The gravity meter is more complicated electronically but is slightly easier to use, short period movements are not so troublesome as with the pendulum apparatus, and it is a great convenience to have a value of gravity almost immediately available.

Professor C. Tsuboi described a gravity meter that he and T. Tomoda are developing. A mass is suspended from a bifilar suspension which is maintained electrically

in rotational oscillations, the period of which depends on gravity. The difference of the period from that of a signal derived from a quartz crystal oscillator is recorded continuously. The mass is suspended on a weak spring and is highly damped to have a period of 50s; in this way the effects of vertical oscillations are considerably reduced. The damper uses silicone oil and an elegant double system is employed to eliminate the effects of surface tension (which depend on temperature). The instrument is suspended in gimbals with a period of 20s and damped by a force proportional to the rate of change of acceleration; the gravity meter then remains closely vertical and there is no correction for horizontal accelerations. Temperature affects the readings only through the linear expansion of the parts and it is sufficient to keep the temperature constant to 0·5 deg C. A single observation takes 50 min.

Professor Tsuboi pointed out that the ship must sail a very straight course to avoid a steady centrifugal force; for the usual speeds of ships the radius of curvature must be greater than 2 km. In a test cruise in a bay, around which gravity is well known, the measured values agreed with interpolated values to 3 mgal.

Mr L. La Coste described the gravity meter that he has developed. The main problems, he said, in constructing a sea gravity meter were to obtain a very small drift and he had achieved $\frac{1}{2}$ mgal per month; the problem of a the large motion which led to hysteresis of the spring if its dimensions changed too greatly; and the problem of averaging the reading. In his instrument the auxiliary force was continually adjusted by hand to keep the spring deviation within a small range and a special computer, using an analogue of the oscillating system, calculated the interval over which the auxiliary force had to be averaged. Over a 30 min averaging time the error due to the ship's motion amounts to 0·5 mgal. A special computer calculated the correction for horizontal accelerations and subtracts it from the reading of the auxilliary force, the difference being continuously recorded. The meter is supported in gimbals stabilized by a servomechanism. The first meter had a range of 6000 mgal but one with a range of 20,000 mgal has now been made. The second-order correction for horizontal accelerations of this magnitude is 100 mgal and it is doubtful if the computer can calculate such a large correction sufficiently accurately. *Mr La Coste* is hoping to use his meter on a surface vessel with a stabilized platform.

Mr A. L. McCahan emphasized the great importance of navigation, the present uncertainties corresponding to much greater errors than those of existing measurements of gravity.

Dr R. L. G. Gilbert discussed some general points. All the instruments that had been described had uncertainties of one to two milligals and in developing a gravity meter for use at sea, an accuracy of one milligal should be aimed at. The natural period of a gravity meter should be far from likely periods of waves to avoid possible resonance effects—only Professor Tsuboi's meter met this condition. Although convenient, an instrument which gave its reading quickly was not essential. The drift should be very small or strictly linear and because of the shortage of submarines, a gravity meter should work on the surface. Finally, the magnitude of the second-order corrections encountered on surface ships made it necessary to use a digital computer, not an analogue one, to achieve the necessary accuracy.

Symposium on Palaeomagnetism and Secular Variation

K. M. Creer

This symposium of the *International Association of Geomagnetism* was held in three sessions, two of which were devoted largely to palaeomagnetism, and one to secular variation. *Sir Edward Bullard* took the chair at the first two sessions and *Dr John W. Graham* at the last.

Palaeomagnetism

About two-thirds of the papers presented were concerned with recent research work in palaeomagnetism, which has developed greatly since the previous I.U.G.G. meeting at Rome three years ago. Then the main topic for discussion was inverse magnetization of rocks. Although the origin of this phenomenon is still somewhat uncertain the probability of reversals of the main geomagnetic field is now admitted by most workers in geomagnetism. The only paper dealing specifically with this problem was one by F. Frölich of Potsdam who, after describing certain heating experiments on normally and reversely magnetized rock samples, concluded that the interpretation will require thorough investigation of the lattice arrangement and state of stress of the materials concerned.

Many other workers reported further occurrences of reversals of magnetization in both sedimentary and igneous formations from the northern and southern hemispheres. It now seems certain that reversals are not local phenomena; for the Quaternary for example, in a paper which was read in their absence, *T. Nagata* and colleagues at Tokyo University reported the occurrence of reversals in Japan; *E. Irving and R. Green*, of the Australian National University reported reversals in the Newer Volcanic Series of Victoria and *K. M. Creer* of the University of Durham reported reversals in Quaternary basalts of Neuquén province, Argentina. World wide occurrence of reversals in geologically contemporaneous formations strongly suggests an origin closely connected with the main geomagnetic field.

Reversals in Tertiary rocks were described by various workers, a most interesting example being given by *A. G. Kalashnikov*, *M. A. Grabovsky* and *G. N. Petrova* of the U.S.S.R. who reported a series of reversals in a Tertiary formation in South East Russia. One reversal occurred within a bed of red limestone, from which a series of samples was taken at different stratigraphical levels. Two consecutive samples less than one foot apart showed opposed magnetization.

Fortunately the lack of a rigorous solution of the problem of inverse magnetization has not hampered progress in the subject. Although reversals are common, there is no experimental evidence to suggest that they are not always through approximately 180° if precautions have been taken to remove unstable components. Thus the axis of the dipole field is unaffected.

The hypothesis of a geomagnetic field which has been dipolar and axial throughout geological time has been fundamental to the latest developments of the subject.

At the 1954 symposium, K. M. Creer, E. Irving & S. K. Runcorn (1954, 1957) presented a paper in which unexpected yet well grouped directions of magnetization of sedimentary and igneous rocks from the British Isles were reported. The rocks ranged in age from late Pre-Cambrian to Tertiary. These directions of magnetization differed significantly from one geological period to the next and they were interpreted as being due to a gradual movement of the average position of the palaeomagnetic pole at a rate of about 0·3° per million years. Since then many more rock formations from other parts of the world have been studied.

E. Irving and R. Green (1958) described data for Australian rocks in a paper entitled "Movement of the Pole relative to Australia since the Proterozoic". The position of the Quaternary palaeomagnetic pole was found not to be significantly different from the present geographic pole. This result agrees with work of J. Hospers (1954) on Quaternary and Upper Tertiary basaltic lava flows in Iceland, and K. M. Creer (1958) found a similar agreement between the palaeomagnetic pole deduced from Argentinian Quaternary basalts and the present geographical pole. This illustrates the average dipolar nature of the geomagnetic field during the Quaternary. For earlier times the Australian pole path is about 160° of longitude away from those calculated for European and North, American rocks, the former path passing through what is now the Atlantic Ocean while the latter pass through regions which are now the Western Pacific Eastern China and Japan. The palaeomagnetic data is the good agreement with palaeoclimatic evidence, in particular the palaeomagnetic dip is steep in the late Proterozoic, Carboniferous and Permian when Australia was glaciated.

In a paper entitled "Palaeomagnetism of the Deccan Traps", *E. R. Deutsch, P. W. Sahasrabudhe* and *C. Radakrishnamurty* described measurements which show that this igneous formation, generally believed to be about 70 million years old, is permanently magnetized along an axis oblique to the present field direction. Their main conclusion was that India has drifted over 50° of latitude and has rotated 25° anti-clockwise relative to the poles since these rocks cooled. In discussion, *J. A. Clegg* suggested that it may be possible to detect a movement of India during the period when the traps were extruded, particularly if a method can be evolved for radioactively dating the flows, which cannot be arranged in chronological order from geological evidence alone. A complication arises however because those flows having steepest palaeomagnetic dips are all reversely magnetized and those with shallower palaeomagnetic dips normally magnetized. This is precisely the effect to be expected if a viscous magnetization (Thellier & Rambert 1954, Creer 1957b,) directed along the present field were superimposed on primary normal and reversed directions which had equal but opposite dips.

K. M. Creer described results from the lavas and baked sandstones of the Serra Geral Formation (Late Triassic to Jurassic) from southern Brazil and Uruguay. These results do not agree with data for Africa published by Nairn (1957) for the Karroo Formation which is geologically contemporaneous with the Serra Geral, and suggest an anti-clockwise rotation of 22° of Africa relative to South America since the Jurassic.

In a paper entitled "Palaeomagnetic Results from Five Continents", *K. M. Creer, E. Irving, R. Green, A. E. M. Nairn and S. K. Runcorn* compared palaeomagnetic measurements they have made in Europe, N. and S. America, Australia and Africa. It was established that movement of the pole relative to each of these continents has occurred during geological time. However, the polar paths are not

identical and it is necessary to allow another degree of freedom by invoking the hypothesis of continental drift in order to explain the palaeomagnetic data adequately. In particular the pole positions derived from N. American rocks are systematically to the west of poles derived from measurements of British rocks of the same age (Creer, Irving & Runcorn 1957). Good agreement of these pole positions is achieved by postulating that N. America and Europe were closer in pre-Cretaceous times. 25° of longitude was suggested although a relative movement of as much as 45° has been estimated by du Bois (1956). It seems necessary to invoke much greater relative continental displacements to reconcile the data from various parts of the Southern hemisphere, but many more measurements, particularly from S. America, Africa and India and if possible from Antarctica, are essential before the general pattern becomes apparent.

Recent palaeomagnetic studies carried out at Imperial College were described by *J. A. Clegg*. It was argued that baked sediments give extremely reliable information about the ancient geomagnetic field because the origin of the natural magnetization is known to be thermoremanent, because its direction can be compared with that of the lava which baked the sediments and because, while the magnetization of lavas might occasionally be affected by viscous flow below the Curie temperature, this can never be so for baked sediments. However, the direction of magnetization of certain Carboniferous baked sediments and igneous rocks (Clegg, Deutsch, Everitt & Stubbs 1957) was noted to be different by about 50° to 60° from directions for other Carboniferous rocks studied by J. C. Belshé (1957) of Cambridge University. There are various possible explanations for this discrepancy, but the most plausible seems to be that since these rocks are from different levels of the Carboniferous a substantial amount of polar wandering must have occurred during those times. This is not inconsistent with the data given by Creer, Irving & Runcorn (1957), for their curves showing a slow but progressive movement of the pole were deduced by methods which involve the smoothing out of changes within each geological period. For instance, data for the Old Red Sandstone described by Creer (1957a) show occasional, yet well-defined directions of magnetization differing by up to 60° from the mean.

M. G. Rutten, of the State University of Utrecht, described measurements undertaken with the aid of co-workers (Rutten, van Everdingen & Zijderveld 1957), of the palaeomagnetism of trachyandesites and basalts of the Oslo Graben and of andesites of the Estérel in S.E. France. Both formations are Permian. Rutten pointed out that the former, sampled from 15 flows, indicated a pole in close agreement with that found by Creer (1957a) for the Exeter Volcanic Series of S.W. England. The results from the Estérel however gave a pole nearer those found by Runcorn (1956), Doell (1955) and Graham (1955) for Permian rocks from the United States. It should be noted, however, that all the samples from Estérel came from a single rhyolite flow. Since this would cool through the Curie temperature of its constituent minerals in a time short compared with that required to average out secular variation, the disagreement of the Estérel pole with that of the Exeter Traps is not surprising and the agreement with the American poles probably fortuitous.

The potential importance of rock magnetism as an aid in the solution of certain geological problems was illustrated by *P. M. DuBois* (1956) of the Geological Survey of Canada. He described the construction of a tentative geological column for the Late Pre-Cambrian of America and Europe from palaeomagnetic data.

Such correlation is extremely difficult or impossible from geological evidence alone but if this is used in conjunction with palaeomagnetic and radiaoctive data it may prove possible to make important advances in our knowledge of Pre-Cambrian stratigraphy.

Four papers were read concerning laboratory studies. *R. F. King* (1957) of the University of Birmingham discussed the effect of grain size, dip of bedding planes and bottom currents during the artificial deposition of Quaternary varved sediments. He concluded that, on occasions, these variables can cause random and systematic errors greater than those of sampling and measurement but also noted that field experiments have shown that such errors appear to be less serious in natural than in artificial deposits. Fortunately, much of the evidence in favour of polar wandering and continental drift is based on results from rocks which have chemical or thermoremanent magnetizations, which are not subject to these errors. *K. M. Creer* (1958) described experiments in which a soft unstable component of magnetization in basaltic lava flows of both Jurassic and Quaternary age from Uruguay and Argentina respectively was removed by treatment in an alternating magnetic field of about 300 oersted peak value. After " cleaning " the precision of the measurements was greatly improved and the directions fell into well-defined normal and reversed groups.

Investigations of the intensity of the ancient geomagnetic field present greater difficulties than those of its direction and comparatively little work has been done in this branch of the subject. *E. and O. Thellier* presented a paper entitled " Researches on the Intensity of the Earth's Magnetic Field in the Past " in which they described various approaches to the problem and discussed some of the difficulties. One cause of error is due to the spontaneous evolution of minerals in the course of time and also to reheating. Results obtained by the study of baked clays were given for the historic period and for more remote periods by the study of volcanic rocks, baked clays and Quaternary metamorphic clays.

In recent papers (Graham 1956, Graham, Buddington & Balsley 1957) much importance has been attached to the role of magnetostriction in the magnetization of rocks. *J. W. Graham* of the Carnegie Institute of Washington described experiments in which non-hydrostatic stresses acting on certain rock samples from the Adirondack metamorphic complex produced permanent changes in magnetization. It was noted that the changes were much greater for rocks containing magnetite or titanomagnetites than for rocks containing minerals from the system $FeTiO_3 - Fe_2O_3$. However, if magnetostriction has played more than a minor role in the magnetization of those rock types from which most of the palaeomagnetic data has been accumulated it would be surprising, considering the basically coherent results obtained from many field observations from many rocks by many different observers. For example, the agreement in pole positions calculated from sediments (containing haematite) and lavas (containing magnetite) of Triassic age from the United States (Banks, duBois, Irving, Opdyke & Runcorn 1957) would indeed be hard to explain if magnetostriction had been important.

Although palaeomagnetic evidence in favour of polar wandering and continental displacement is now abundant, the origin of these phenomena is not understood. *W. S. Jardetsky* of Manhatten College, New York, discussed the relationship between the break up of a single primeval continent and polar wandering. He stressed the necessity of separating polar wandering due to displacements of the axis of rotation of the earth in space from that due to gliding of the solid shell over

a fluid core. Convection currents in the mantle have frequently been advocated as a possible cause of continental displacement. *A. A. de Mendoza Dias* reviewed magnetic and seismic evidence from near the Azores which he considered in accord with a convective system in the mantle, the direction of flow being upwards beneath Africa, horizontal in a N.W. direction towards the Azores, where it descends on meeting another convection current from the N.W. The latter, he thought, might possibly be related to the origin of the Mid-Atlantic ridge.

Secular variation

The final session was devoted mainly to papers concerning theories of the secular variation.

F. J. Lowes of the University of Durham and *A. Herzenberg* of the University of Manchester (Lowes & Herzenberg 1958) described developments in the "eddy" model which was first proposed by Sir Edward Bullard (1948). The non-dipole field and the secular variation are supposed to be due to electromagnetic induction in motions (probably convective) of the conducting fluid of the Earth's core. Assuming the convective patterns are in the form of eddies, considerations of the available power and an electromagnetic saturation effect show that the diameter of the eddies must be several hundreds of kilometers. The effect of stationary conducting material surrounding the eddies and the boundary of the core are allowed for.

The observed rate of growth of the secular variation field is so rapid that, assuming any reasonable value for the electrical conductivity of the core, the skin depth is but a small fraction of the size of the eddies. *P. H. Roberts* of the University of Durham, in a paper (Roberts 1958) which dealt with the propagation of induced fields within the Earth's core, showed that the dilemma could be overcome because the core is known to be liquid, in which case disturbances in it must be propagated by hydromagnetic waves. These are much more effective in the propagation of disturbances over large distances than electromagnetic diffusion, and it was shown that the hydromagnetic skin-depth greatly exceeds the diameter of the core.

Sir Edward Bullard and *D. W. Allen* of the University of Cambridge discussed an alternative mechanism in which the upward streaming motions of convection currents within the core drag lines of force of the toroidal field from the interior to the surface. They deduced that the magnitude of the effect is sufficient to account for the secular variation.

S. K. Runcorn of the University of Durham read a paper on "The Theory of the Geomagnetic Secular Variation" in which he pointed out thas previously it has been assumed that the coincidence of the mean geomagnetic axit and the geographical axis reflects the dominance of the coriolis forces in the Earth's core. However, he showed that this coincidence is bound to result even if it were possible for the magnetic field generating in the core to have a permanent non-axial component. This result arises from the fact that the Earth's core and mantle rotate at different velocities. Thus at a point fixed to the Earth's surface non-axial components of the magnetic field are averaged out over times long compared to the period of relative rotation of core and mantle. This relative rotation of the core and mantle was discovered by the observation of the westward turn of the geomagnetic field and is a consequence of the weak coupling between the core and mantle. It can be shown from observations in historical times that the geo-magnetic field vector at most points on the Earth's surface tends to describe a

clockwise rotation in space viewing along the vector to its north seeking pole. It turns out that the clockwise sense of the rotation is produced by the westward rotation of the lower harmonics of the geomagnetic field.

S. Barta of Budapest summarized his ideas concerning the origin of the secular variation and deduced the existence of a circular current of $10^7 - 10^8$ amps and radius 3,000 km at a depth of 3,000 km beneath Pakistan. A precession of the axis of this eddy was suggested and this might be the cause of the observed movement of about 10 km/yr of the centre of the anomaly.

The main geomagnetic field

F. J. Lowes read a report by *A. Herzenberg* in which it is proved possible to postulate a pattern of motions in a homogeneous sphere of isotropic conductive material in such a way that the whole acts as a steady dynamo producing an external field. Though it is not suggested that the motions in the Earth's core have the simple form assumed the proof itself is rigorous and provides an existence theorem.

Anselmo Chargoy of the Universidad de Mexico described how the central quadrupole could be represented by two eccentric coplanar current loops. If the current strengths varied with respect to each other a dipole varying in sense and intensity as well as a quadrupole would be observed on the Earth's surface.

Most of this papers read at this symposium are in course of publication in *Annales de Géophysique*. Thanks are due to Mr E. Irving, Dr F. J. Lowes and Dr P. H. Roberts for helpful discussion during the preparation of this review.

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Symposium on Fault Plane Conditions

The meeting of the *International Association of Seismology and Physics of the Earth's Interior* on September 12th was a symposium on fault plane conditions organized by *Dr J. H. Hodgson* of the Dominion Observatory, Ottawa.

The first paper was read by *Professor Perry Byerly* who discussed the longitudinal and transverse displacements due to specified disturbances in a homogeneous infinite medium. In such circumstances, the nature of the disturbance can be inferred from properly distributed seismometers, using the signs of P and S at first emergence and the ratios of the amplitudes of P and S. It was generally found that a couple at the source reproduced conditions most successfully. *Professor Byerly* remarked that American seismologists, who had much experience of long surface faults with horizontal slip, often thought in terms of simple couples, whereas Japanese seismologists more often considered magma injection with tension in one direction and compression in a perpendicular direction. If the source function is a couple, S and P have nodes on the fault plane. P also vanishes on the plane perpendicular to the fault plane and the direction of motion, the auxiliary plane. There are, in addition, relations between S_H and S_V. However, the observed relations between amplitudes are more complicated than predicted by the theory and the practical procedure is to plot the signs of P, S_V and S_H and to determine the traces of the nodal planes as those curves across which the signs change; one of these planes is the fault plane, the other the auxiliary plane. In this way, the dip of the fault plane may be found.

Dr Keylis-Borok described recent Russian investigations of earthquake mechanisms, giving most attention to studies of the effects of intermediate boundaries and showing some of the extensive sets of graphs and nomograms that he has constructed to show the nodal lines for various source and boundary conditions. One of the difficulties of interpretation is that the fault plane is itself a boundary; in addition, there are usually other boundaries that lead to results different from those to be expected in a homogeneous medium. In particular, if there is a boundary near the source, there may be additional nodal planes for P and these can lead to difficulties in interpretation.

Dr Keylis-Borok then described the results of studies of earthquakes in the U.S.S.R., pointing out that these had been made before the foregoing theoretical work, which was very recent, was available. The principal results were from the N.W. Pacific, the Hindu Kush, Ashkhabad, Garm, the Tien Shan and the Caucasus. In over three hundred earthquakes that had been studied, the motion could usually be interpreted as due to a couple. The general features that had been found were that there were distinct zones with a common type of dislocation, usually characterized by two strike directions. These zones were associated with tectonic complexes but the directions were not correlated with local structures near the epicentre. Where there was a tectonic depression, the motion was such as to raise the depressed zone. This had also been found from geodetic observations on earthquakes in Java.

Dr Press described some ultrasonic model experiments on radiation from faults.

The initial and boundary conditions met with in practice were too complex for theoretical treatment, and model experiments were therefore valuable, although it had to be realized that a true model cannot yet be made; for instance, the speed of rupture cannot be reproduced nor the finite extent of the source.

The experiments were made on a disk of plexiglass twenty inches in diameter and one-sixteenth of an inch thick. It was excited at the centre by barium titanute transducers and the *P* and *S* waves were detected by separate transducers at the rim and were recorded on a cathode ray oscilloscope which was photographed. With a simple linear displacement at the centre, the *P*-wave had a maximum in line with the direction of motion and nodes at right-angles to it; the *S*-wave, on the other hand, had nodes in the direction of motion and maxima at right-angles to it. With dipole sources, there were *P*-wave nodes on the axis of the dipole and at right-angles to it with maxima at 45° to the axis; the *S*-wave nodes were on the axis of the dipole and the maxima at right-angles to it. These results are to be expected from the theory of an infinite homogeneous medium. In a further series of experiments, a very narrow slit was cut in the centre of the disk, parallel to the imposed displacement, and was filled with silicone grease to maintain continuity of normal stress. It was made as narrow as possible (the width should be small compared with the thickness of the disk) but even so it was about one wavelength wide. With a dipole source, the *P*-wave pattern is unaffected by the presence of the slit, but with *S*-waves, there is an additional anti-node on the axis of the slit and dipole, the nodes being in the neighbourhood of 45° of the axis. The results are not critically dependent on the length of the slit. If instead of filling the slit with a material without rigidity it is filled with one of lower rigidity than the plexiglass, the results are essentially the same. *Dr Press* concludes from these experiments that solutions for fault plane conditions which are based on *S*-wave results may be erroneous but that results based on *P*-waves should be more reliable.

In the discussion, *Professor Coulomb* suggested that the *S*-wave patterns found by Dr Press were due to diffraction at the end of the slit and Dr Press agreed that this was so.

Dr J. H. Hodgson discussed the use of the Null Vector, as he terms it, as a guide to tectonic patterns. From the pattern of the signs of *P* displacements, two planes may be constructed, either of which may be the fault plane containing the direction of displacement, the other being the perpendicular auxiliary plane. The intersections of these planes with the Earth's surface are circles and the line in which the planes intersect passes through the points in which the circles intersect. Whichever of the planes is the fault plane, this line is unique, and it is this line that is the Null Vector. In an earlier analysis of seventy-five earthquakes, *Dr Hodgson* found that sixty-seven had strike-slip faulting. He had now analysed the data for eighty-five further earthquakes, excluding near-station observations and solutions based on *S*-wave data, and he finds that in these strike-slip faulting predominates in the circum-Pacific belt and in New Guinea. In the New Zealand-Fiji area, for earthquakes at all depths from 0 to 700 km, the Null Vector is within 3° of N–27E, a direction corresponding with tectonic features. There are similar correspondences in the New Hebrides and South America, and *Dr Hodgson* suggested that the Null Vector might be useful in elucidating tectonic patterns. In the Aleutian Islands and Alaska, well-defined solutions are obtained, but there is no clear overall pattern in the Null Vector although neighbouring epicentres have Null Vectors with similar directions.

Professor Benioff began his discussion of circum-Pacific tectonics by considering

whether the pattern of major earthquakes was not more consistent than that of minor ones. Thus the San Andreas fault system is a major break in the Earth's crust, running from Oregon to the Gulf of California, which in the 1906 earthquake broke the surface for 600 km. The major movements on this fault are related to the tectonic pattern whereas the minor movements resolve incompatibilities in the major movements and may not be directly related to the tectonics. As instances of other major systems, *Professor Benioff* gave Kamchatka, where there is right-handed slip on a fault segment of 1000 km, and the zone of recent earthquakes in the Aleutians, again about 1000 km long. Fault plane solutions for the Aleutians are not yet available but the geology indicates right-handed strike-slip faulting. Around the Pacific the predominant fault movement is right-handed strike-slip faulting as in California, Alaska, northern Canada, the Aleutians, Japan, Kamchatka and New Zealand. The evidence for South America is inconclusive but the general picture from the whole circum-Pacific zone is that the ocean is moving counter-clockwise while radial motions effect consequent local readjustments. The radial movements were of the margins towards the oceanic mass and were responsible for the oceanic deeps and associated mountain ranges.

Professor Benioff said that there were still some inconsistencies in the evidence. Thus, two *G*-waves, G_2 and G_1 had been received in California from Kamchatka earthquakes, having left the source in opposite directions to travel on the opposite sides of the Earth. However, they indicated that the direction of motion was the same on both sides of the fault.

Professor D. B. McIntyre gave an account of the criticisms of Dr Hodgson's work that he had been developing and discussed the kinematics of faulting as inferred from seismic data. These data consist of distributions of the signs of the first movement of the *P* phase from which the orientations of the principal and auxiliary planes are not in general uniquely determined. The line of intersection of these planes—Dr Hodgson's Null Vector—is, however, well defined. Corresponding to the system of axes used in structural geology, axes *a*, *b* and *c* may be chosen with reference to a fault motion, *b* being in the fault plane at right-angles to the direction of motion, *a* being in the fault plane parallel to the direction of motion, and *c* being perpendicular to the fault plane. There is just one plane of symmetry. The *b* axis corresponds to the Null Vector: but *Professor McIntyre* deprecates the use of the term " Null Vector " since what is in fact meant is a direction and not a true vector. In analysing the results of the earthquake studies, the first question to decide is whether the *b* axes of a group of foci are all nearly parallel and *Professor McIntyre* considers that Dr Hodgson's method of plotting the results is not most suitable for this; instead he plots the *b*-axes of the foci in an area on a stereographic diagram. The second question is, if the *b*-axes are all nearly parallel, what is the significance of this fact? Now there is another set of directions associated with a group of foci, namely the intersections of the fault planes, and if these intersections cluster strongly, they define a unique direction, the β-direction. Kinematical and dynamical considerations make it most likely that the β-direction should coincide with the *B* kinematic axis (the upper case letter indicates that the *b* directions for different earthquakes in the region are all nearly the same). This consideration might be used to differentiate between the fault plane and auxiliary plane, for if the choice is made in one way it might be expected that the β and *B* directions would coincide, but not if the other choice is made.

This situation occurs in the south-west Pacific region from New Zealand to

Fiji. The *b*-axes cluster strongly defining a *B* direction, thus showing that strike-slip faulting prevails. The poles of the nodal planes either lie on the plane perpendicular to *B* or on a line at right-angles to it and if the nodal planes parallel to the physiographic features are chosen as the fault planes, there is a strong β-maximum coinciding with the *B*-direction. This therefore seems to be the correct choice; the faulting is strike-slip. The dynamic interpretation cannot, however, be straightforward because half the faults in this area have right lateral motion and half left; there is a similar result for Indonesia. The direction of motion in the New-Zealand–Fiji region is at right-angles to the Kermadec and Tonga trenches in the sense of increasing the offset between them. It seems, in conclusion, that in order to resolve the ambiguity of first motion data, the distribution of the kinematic axes must be such that it is possible to show that the β and *B* directions coincide for one assignment of planes and not for the other.

Dr P. St. Amand reviewed the evidence about circum-Pacific orogeny which, he said, in the north-west Pacific region involved mainly right lateral faulting. The San Andreas fault is only one element of a wide zone which extends far inland, and *Dr St. Amand* described in some detail the very complex pattern of faults in the Garloch region which was due to overall shear. In Alaska, geological evidence shows that there have been right lateral movements of up to 150 miles. Geological evidence shows that around the Pacific there has been counter-clockwise rotational movement since the Mesozoic period. *Dr St. Amand* suggests that the cause of this movement may be a convection current cell centred under the Pacific, the consequent Coriolis force causing the observed rotation.

Public Lectures

Two evening lectures, open to the general public, were given during the period of the Assembly. *E. I. Tolstikov*, the deputy head of the Central Department for the Arctic Seaway of the U.S.S.R., speaking on " Polar Research ", described current Russian investigations in the Arctic, and *Professor L. V. Berkner*, the President of the International Council of Scientific Unions, in a lecture with the title " Instrumented Earth Satellite ", discussed the problems of launching a satellite and the observations that it was intended to make with the American satellite vehicles. Possible experiments range from simple observations of the orbit, from which the Earth's ellipticity and the drag of the atmosphere may be inferred, to the installation of a caesium frequency standard in the satellite in order to measure the gravitational displacement of the resonant frequency predicted by general relativity.

The most significant immediate advances, however, are likely to be in meteorology where, for the first time, it will be possible to obtain data essential to quantitative studies. Considering the atmosphere as a heat engine, the satellite will enable the heat input to be measured for the first time, both its total amount and its distribution over the Earth. It will be possible to measure the energy input from the Sun in different wavebands and it will be possible to measure the energy reflected (the albedo of the Earth). By studying the distribution of the net input over the Earth it should be possible to understand why the heat engine works more vigorously in some places than others. The albedo measurements should also be of immediate value in weather forecasting since it should be possible to deduce from them the distribution of clouds over the otherwise inaccessible oceans and to observe the changes in the distribution.

Rocket experiments have shown that the Sun emits X-rays which vary very greatly in intensity and it is hoped that these also will be studied as well as cosmic rays. Observations will be made of the Earth's magnetic field, one of the main objects being to investigate the ring current postulated by Chapman and Størmer to exist at great distances, and possibly connected with the origin of whistlers.

Resolutions on Deep Drilling

Of the resolutions adopted by the General Assembly of the I.U.G.G. at its second plenary session, the two which attracted most attention proposed that attempts should be made to drill very deep bore-holes through the crust of the Earth. The significance of these resolutions stems in part from the fact that the I.G.Y. artificial satellite programmes were started as a result of resolutions passed at the Rome Assembly of the I.U.G.G. The resolutions recommended:

(1) that an investigation should be made of the possibility of drilling to the Mohorovičic discontinuity below the ocean;

(2) that a boring should be made through the sediments below the ocean.

The Mohorovičic discontinuity is about 5–10 km below the sea-bed of the deep oceans, and it should be possible to reach it by drilling 10–15 km from an island or atoll platform. There is some danger that the island will have a root, with a consequent depression of the layer with the seismic velocity of 8·1 km/s, but the Mohorovičic discontinuity should certainly be much shallower than below continents. The shortest drilling path would be obtained by drilling from an artificial platform in the ocean, but the cost of this would probably outweigh the advantage gained. It would evidently be most valuable to learn how rapid the transition is at the Mohorovičic discontinuity and what the constitution of the upper mantle is.

Bore-holes have already been drilled by oil companies to over 7 km and there appears to be no insuperable difficulty in doubling this depth. The rock to be drilled would be hard basalt, through which penetration is slow by conventional means. However, mining companies do have experience of deep bores in this type of material, and there is an advantage in that hard rock will support itself and will not need very complicated drilling-mud treatment. There is scope in this project for trying out new methods, such as burning or blasting the rock. In any case the bore-hole will probably take some years to complete, and effort will be well spent in preliminary research.

The second project is feasible today, according to the oil company experts who have been taking cores off the Californian coast for the past five years. The problem is to drill through about a thousand feet of soft sea-bed sediment, and to bring to the surface samples of this sediment and of the hard rock which seismic experiments show to lie beneath. A 300 ft hard rock core has been collected in 1500 ft of water in the Pacific, by drilling with a rotary rig mounted on a barge which is anchored. The core samples are pulled up the inside of the hollow drill-pipe by a tube and a wire-line. In 20,000 ft of water the barge could ride freely, keeping its position with reference to a buoy anchored on piano-wire. The long drill-pipe would allow all the flexibility needed. The operation would be a one-shot affair, since it would be impossible to find the old hole once the drill had been removed. The amount of hole drilled would be determined by the wearing-out of the bit. It should be possible to penetrate the soft sediment and a hundred feet of underlying hard rock.

РЕЗЮМЕДОКЛАДОВ, В ПЕРЕВОДЕ НА РУССКИЙ ЯЗЫК

ПРИМЕНЕНИЕ ТЕОРИИ КОНЕЧНОГО НАТЯЖЕНИЯ К СОСТОЯНИЮ ВНУТРИ ЗЕМНОГО ШАРА.

Дж. Ф. Эверден

Доказывается, что подход Пуанкарэ-Бриллуэна к теории конечного упругого натяжения приводит к уравнениям, равноценным уравнениям полученным методом Мэрнахан-Бирча. Применение уравнения давление-натяжение Бриллуэна к состоянию внутри земного шара (т. е. к моделям Буллена) указывает на возможность нескольких различных выводов. Рассматривается относительная точность полученных параметров и делается вывод, что данные Буллена могут указывать на наличие нижнего слоя коры, для которого $\rho_0=4{,}0$ г/см^3, $K_0/\rho_0=33$ км^2/сек^2 и наружного слоя коры, для которого $\rho_0=6{,}75$ г/см^3, $K_0=1{,}2.10^{12}$ дин/см^2.

ДИФФРАГИРОВАННЫЕ СЕЙСМИЧЕСКИЕ ВОЛНЫ ВБЛИЗИ ОГИБАЮЩЕЙ *PKP*

К. Э. Буллен и Т. Н. Бюрк-Гэффни

Исследовано время пребывания волн P' в интервале $137°<\triangle<142°$, регистрированных после четырех взрывов водородных бомб в 1954 году. Ранние явления, регистрированные в этом интервале, приписываются влиянию диффрагированных волн *PKP* вблизи огибающей при 142°. Эти результаты представляют собой небольшое конечное звено в серии выводов доказывающих существование внутреннего ядра. Рассмотрена также возможность регистрирования диффрагированных волн *PKP* при помощи наблюдения над естественными землетрясениями. Результаты свидетельствуют о том, что сильные управляемые взрывы могут расширить познания о состоянии в глубине земной коры.

КАЛИБРИРОВАНИЕ ПРИБОРОВ ДЛЯ ИЗМЕРЕНИЯ СИЛЫ ЗЕМНОГО ТЯГОТЕНИЯ ПУТЕМ СРАВНЕНИЯ С МАЯТНИКАМИ

А. Х. Кук

Рассмотрена регулировка способом наименьших квадратов ряда наблюдаемых указаний маятников и приборов измерения силы земного тяготения. Выведена общая формула для калибрационного фактора прибора в зависимости от относительной важности наблюдаемых указаний маятников и приборов. Получены также выражения этого фактора в том случае, когда сеть расположения маятников и приборов идентична и когда не производятся излишние наблюдения. Только в этом случае фактор не зависит от относительной важности наблюдаемых указаний маятников и приборов. Предлагается, что, в виду возможности неудовлетворительного определения относительной важности показаний, при производстве наблюдений следует соблюдать последнее условие. Рассмотрены методы обработки взаимно зависимых наблюдений, а также применение результатов для калибрирования приборов измерения силы земного тяготения.

ИЗУЧЕНИЕ НАПРАВЛЕНИЯ МИКРОСЕЙСМИЧЕСКИХ ВОЛН ВОСПРИНИМАЕМЫХ ОБСЕРВАТОРИЕЙ В КЬЮ

Х. М. Айер

Применяется видоизменение корреляционного метода Дарбишира для изучения направлений распространения микросейсмических волн, с целью обследования трех быстро перемещающихся циклонических депрессий в Атлантическом океане, имевших место в октябре 1951 г., в октябре 1952 г и в декабре 1953 г. Вычисленные и истинные положения центров бури хорошо совпадают в том случае, когда депрессия достаточно далеко от берегов и когда метереологическая обстановка не является сложной. С другой стороны, для бури вблизи берега, множество источников микросейсмических явлений затрудняет их изучение. Точность обследования улучшается благодаря фильтрованию микросейсмических волн, использованию узкой полосы у пика спектра и осуществлению записей в течение продолжительного периода. Во всех исследовавшихся случаях микросейсмические волны заключали в себе значительное количество волн Лава. Отношение интенсивности волн Лава к интенсивности волн Релея зависит повидимому от положения и интенсивности центра бури.

ВЫДЕЛЕНИЕ ЭНЕРГИИ ПРИ ЗЕМЛЕТРЯСЕНИЯХ

Леонид Кнопов

Принцип применения теории статической упругости для изучения потери энергии между двумя состояниями одной и той же структуры, был использовзн Старром для решения проблем связанных с распространением трещин в поле сдвига. Этот способ можно приспособить для решения проблемы выделения энергии при обвале, как например для сброса в Сан Андреасе, происходящем в однородной среде подверженной равномерному сдвигающему напряжению. В идеальном виде, свойства такого сброса— растянутая структура по сравнению со своей глубиной и скользящее движение пласта вдоль сброса. Благодаря симметричному расположению, такая конфигурация может быть отражена поверхностью земли так, что проблема может быть сведена к случаю бесконечно длинного, вскрытого сброса в беспредельной, однородной, изотропной и упругой среде. Среда подвергается равномерному сдвигающему напряжению бесконечного действия и подразумевается, что сдвигающее напряжение не распространяется зпределы полосы сдвига. Эта проблема, в плоскости, решается легче при помощи векторного исчисления, нежеи тензорным способом. Таким образом она аналогична, в электричестве, проблеме ленты идеальной проводимости находящейся в однородном электрическом поле, или же проблеме сопротивления рейки находящейся в равномерном гидродинамическом потоке. Отсюда можно вывести распределение напряжения и относительное перемещение внутри среды до и после сброса. В случае землетрясения в Сан Франциско, в 1906 г, этим способом обнаруживается выделение энергии в поле сдвига, с начала до конпа сброса, в 4.10^{23} эрг. Эта величина должна превосходить энергию упругой волны.

ГЕОМАГНИТНЫЕ КОЛЕБАНИЯ И ВНЕШНЯЯ АТМОСФЕРА ЗЕМЛИ

Т. Обаоаши и Дж. А. Джэкобс

Рассмотрены гидромагнитные колебания внешней ионизированной атмосферы Земли по геомагнитным силовымл иниям. Распределение ионной плотности во внешней атмосфере за пределами ионосферы выводится из данных наблюдений над всемирнсй геомагнитной пульсацией. Найдено, что ионная плотность около 10^3 на $см^3$ на расстоянии равном нескольким радиусам земли и экспоненциально убывает до показателя порядка 5 на $см^3$ в междупланетном пространстве. Также теоретически обсуждается состояние температуры внешней ионосферы.

ПЕРЕМЕЩЕНИЕ ПОЛЮСА ОТНОСИТЕЛЬНО АВСТРАЛИИ

Э. Эрвинг и Р. Грин

Во время верхнепротерозойского периода средний геомагнитный полюс повидимому находился вблизи Австралии. Ко времени кембрийского периода он повидимому перекочевал в область нынешней Южной Африки и далее, в район Южного океана ко времени силурийского и раннего девонского периодов. Во время верхнекаменноугольного и пермского периодов, полюс находился в Тасманском море и Австралия снова оказалась в высокой геомагнитной широте. Позднее он медленно переместился к югу, достигнув положения настоящего географического полюса к концу третичного периода.

ИССЛЕДОВАНИЕ СЕЙСМИЧЕСКОГО ПРЕЛОМЛЕНИЯ У СЕВЕРНЫХ БЕРЕГОВ КОРНВАЛЛИСА

А. С. Мерриуедер

Исследование сейсмического преломления звука произведено по четырем направлениям отходящим от гидрофона, расположенного на дне моря вблизи Перанпорта (Корнваллис). В заключение выведены структура и звуковые свойства дна моря и измерено ослабление поверхностных волн в зависимости от расстояния, как в месте исследования, так и при помощи гидрофона опущенного в находящийся неподалеку колодезь. Полученные результаты указывают на то, что дно моря в этой области состоит из трех слоев: песчаного или наносного слоя толщиной в 25–30 м (80–100 фут), в котором скорость сжимающих взрывных волн 1500–1680 м/сек (4950–5500 ф/сек) и слоев имеющих высокую звукопроводимость—скалистого в 397 м (1300 ф) толщиной, в котором скорость звуковых волн 4350 м/сек (14250 ф/сек) и слоя очень большой телщины, в котором скорость звуковых волн 5650 м/сек (18500 ф/сек). Слой, в котором скорость звуковых волн в 4300 м/сек (14000 ф/сек), соответствует по скорости волн сланцам, извлеченным из карьеров около Перанпорта, что указывает на то, что этот слой является северным продолжением палеозойских пород Девоншира и Корнваллиса. Ослабление поверхностных волн было найдено равным приблизительно 12 дб по двойному расстоянию. Полученные результаты сравниваются с существующими теориями распространения взрывных звуковых волн по упругим и текучим слоям морского дна и даются кое какие указания относительно дальнейших исследований.

О ВИДОИЗМЕНЕНИИ ЗАКОНА ЛОМНИЦА О ПОЛЗУЧЕСТИ ГОРНЫХ ПОРОД

Х. Джеффрис

К. Ломниц доказал, что ползучесть горных пород под непрерывным напряжением возрастает логарифмически со временем. Данные, касающиеся вариации высоты сейсмических волн, свидетельствуют о том, что для большей части земной коры предпочтителен термин t^{α}, где α приблизительно равна 0,17. Это правило применяется в данном докладе к перемещению гармонической неровности поверхности. Есть указания на то, что этот род ползучести не является причиной нарушения изостатического равновесия в пределах геологического периода и делается предположение, что деформация пород происходит благодаря излому, или растяжению приближающемуся к пределу упругости.

Geophysical Journal

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1. Authors are advised to follow the general suggestions for the preparation of scientific papers given in *Notes on the preparation of papers to be communicated to the Royal Society* and in *The printing of mathematics* by T. W. Chaundy, P. R. Barrett and C. Batey (Oxford University Press, 1954). In particular the abbreviations for the names of periodicals used in the *Geophysical Journal* are those given in *Notes on the preparation of papers* . . . and the symbols, signs and abbreviations are those recommended for British scientific publications by the Symbols Committee of the Royal Society.

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